Exercise 4.1

(a) $x - 24 = 8$ — $IQR = Q_3 - Q_1$ (M1)

$x = 32$ — (A1)

(b) The number of people

$= (30 + 10)\left(\frac{3}{4}\right)$ — $\frac{3n}{4}$ (M1)

$= 30$ — (A1)

(c) The mean number of hours

$= \frac{900}{30}$ — $\frac{x_1 + x_2 + \cdots + x_n}{n}$ (M1)

$= 30$ — (A1)

(d) (i) The total number of hours

$= (40)(28)$ — $n\overline{x}$ (M1)

$= 1120$ — (A1)

(ii) The mean number of hours

$= \frac{1120 - 900}{10}$ — $\frac{x_1 + x_2 + \cdots + x_n}{n}$ (M1)

$= 22$ — (A1)

(e) (i) The required mean

$= 22 - 3$ — $\overline{x} - k$ (M1)

$= 19$ — (A1)

(ii) The required variance

$= 2.5^2$ — σ^2 (A1)

$= 6.25$ — (A1)

Exercise 4.2

(a) 20 minutes (A1)

(b) 10 minutes (A1)

(c) The number of students

$= 270 - 90$ — $270 - 90$ (M1)

$= 180$ (A1)

(d) The number of students spent not more than k minutes to travel to school

$= 360 - 360 \times \frac{1}{6}$ — $360 - 360 \times \frac{1}{6}$ (M1)

$= 300$ — 300 (A1)

$\therefore k = 30$ (A1)

(e) r

$= 25 + (1.5)(10)$ — $Q_3 + 1.5IQR$ (M1)

$= 40$ (A1)

(f) Systematic sampling (A1)

Exercise 4.3

(a) (i) $r = 0.3346566771$

$r = 0.335$ (A1)

(ii) Weak, positive (A1)(A1)

(b) (i) $a = 0.4228571429$

$a = 0.423$ (A1)

$b = 6.22$ (A1)

(ii) b represents the expected sales in 2023. (A1)

(c) The estimated sales

$= 0.4228571429(2.5) + 6.22$ $x = 2.5$ (M1)

$= 7.277142857$ millions of dollars

$= 7.28$ millions of dollars (A1)

Exercise 4.4

(a) (i) $\text{P}(A \cap B) = 0$ (A1)

(ii) $\text{P}(B) = 0.5$ (A1)

(b) (i) $\text{P}(A \cup C) = \text{P}(A) + \text{P}(C) - \text{P}(A \cap C)$ Setting equation (M1)

$0.33 = 0.05 + \text{P}(C) - 0.02$

$\text{P}(C) = 0.3$ (A1)

(ii) $\text{P}(A \cap C') = 0.03$ (A1)

(iii) $\text{P}(A' \cap C) = 0.28$ (A1)

(c) $\text{P}(A' \mid C)$

$= \dfrac{\text{P}(A' \cap C)}{\text{P}(C)}$ $\dfrac{\text{P}(A' \cap C)}{\text{P}(C)}$ (M1)

$= \dfrac{0.28}{0.3}$

$= \dfrac{14}{15}$ (A1)

(d) $\text{P}(B \cap C)$

$= \text{P}(B)\text{P}(C)$ $\text{P}(B)\text{P}(C)$ (M1)

$= (0.5)(0.3)$

$= 0.15$ (A1)

Exercise 4.5

(a) The required probability

$$= \mathrm{P}(Vaccination)\mathrm{P}(Virus \mid Vaccination) + \mathrm{P}(No\ vaccination)\mathrm{P}(Virus \mid No\ vaccination)$$ Sum of two paths (M1)

$$= (73\%)(0.07) + (1 - 73\%)(0.31)$$ Sum of two paths (A1)

$$= 0.1348$$ (A1)

(b) The required probability

$$= \frac{\mathrm{P}(No\ vaccination)\mathrm{P}(Virus \mid No\ vaccination)}{\mathrm{P}(Vaccination)\mathrm{P}(Virus \mid Vaccination) + \mathrm{P}(No\ vaccination)\mathrm{P}(Virus \mid No\ vaccination)}$$ Bayes' theorem (M1)

$$= \frac{(1-73\%)(0.31)}{0.1348}$$ $\frac{(1-73\%)(0.31)}{0.1348}$ (A1)

$$= 0.620919881$$

$$= 0.621$$ (A1)

Exercise 4.6

(a) $\mathrm{P}(X=0)+\mathrm{P}(X=10)+\mathrm{P}(X=20)+\mathrm{P}(X=30)=1$ Sum of probabilities (M1)

$5k+5k+2k+8k=1$

$20k=1$

$k=0.05$ (A1)

(b) $\mathrm{E}(X)$

$=(0)(0.25)+(10)(0.25)+(20)(0.1)+(30)(0.4)$ $(0)(0.25)+\cdots+(30)(0.4)$ (A1)

$=16.5$ (A1)

(c) $\mathrm{Var}(X)$

$=\mathrm{E}(X^2)-(\mathrm{E}(X))^2$ $\mathrm{E}(X^2)-(\mathrm{E}(X))^2$ (M1)

$=(0^2)(0.25)+(10^2)(0.25)+(20^2)(0.1)$ $(0^2)(0.25)+\cdots+(30^2)(0.4)$ (A1)

$+(30^2)(0.4)-16.5^2$

$=425-272.25$

$=152.75$ (A1)

Exercise 4.7

(a) Let $X \sim \mathrm{B}(75, 0.11)$ be the number of left-handed students.

$\mathrm{E}(X)$

$= (75)(0.11)$ — np (M1)

$= 8.25$ — (A1)

(b) The required probability

$= \mathrm{P}(X = 10)$ — $\mathrm{P}(X = 10)$ (M1)

$= 0.1103559416$

$= 0.110$ — (A1)

(c) The required probability

$= \mathrm{P}(X > 6)$ — $\mathrm{P}(X > 6)$ (M1)

$= 1 - \mathrm{P}(X \le 6)$ — $1 - \mathrm{P}(X \le 6)$ (M1)

$= 0.7312259865$

$= 0.731$ — (A1)

(d) The required probability

$= \mathrm{P}(X = 10 \mid X > 6)$ — $\mathrm{P}(X = 10 \mid X > 6)$ (M1)

$= \dfrac{\mathrm{P}(X = 10 \cap X > 6)}{\mathrm{P}(X > 6)}$

$= \dfrac{\mathrm{P}(X = 10)}{\mathrm{P}(X > 6)}$ — $\dfrac{\mathrm{P}(X = 10)}{\mathrm{P}(X > 6)}$ (M1)

$= \dfrac{0.1103559416}{0.7312259865}$

$= 0.1509190642$

$= 0.151$ — (A1)

Exercise 4.8

Let X be the time taken for students to travel to school.

$\mathrm{P}(X<11)=0.39$

$\mathrm{P}\left(Z<\dfrac{11-\mu}{\sigma}\right)=0.39$ — $Z=\dfrac{X-\mu}{\sigma}$ (M1)

$\dfrac{11-\mu}{\sigma}=-0.279319035$ — -0.279319035 (A1)

$11-\mu=-0.279319035\sigma$

$\mu=11+0.279319035\sigma\ldots(1)$ — $11+0.279319035\sigma$ (A1)

$\mathrm{P}(X>13)=0.12$

$\mathrm{P}\left(Z>\dfrac{13-\mu}{\sigma}\right)=0.12$

$\dfrac{13-\mu}{\sigma}=1.174986791$ — 1.174986791 (A1)

$13-\mu=1.174986791\sigma$

$\mu=13-1.174986791\sigma\ldots(2)$ — $13-1.174986791\sigma$ (A1)

$(1)=(2)$

$11+0.279319035\sigma=13-1.174986791\sigma$ — Setting equation (M1)

$1.454305826\sigma=2$

$\sigma=1.375226561$

μ

$=11+0.279319035(1.375226561)$

$=11.38412696$

$\therefore$ $\therefore\ \mu=11.4$, $\sigma=1.38$ — (A1)(A1)

Exercise 4.9

(a) (i) Let H be the height of a tree.

The required probability

$= \text{P}(H < 3.2)$ $\text{P}(H < 3.2)$ (M1)

$= 0.0111354575$

$= 0.0111$ (A1)

(ii) The required probability

$= \text{P}(H < 3 \mid H < 3.2)$ $\text{P}(H < 3 \mid H < 3.2)$ (M1)

$= \dfrac{\text{P}(H < 3 \cap H < 3.2)}{\text{P}(H < 3.2)}$

$= \dfrac{\text{P}(H < 3)}{\text{P}(H < 3.2)}$ $\dfrac{\text{P}(H < 3)}{\text{P}(H < 3.2)}$ (M1)

$= \dfrac{0.0021374316}{0.0111354575}$

$= 0.1919482518$

$= 0.192$ (A1)

(b) $\text{P}(H > \alpha) = 0.31$ $\text{P}(H > \alpha)$ (M1)

$\alpha = 4.173547621$

$\alpha = 4.17$ (A1)

(c) The required probability

$= (\text{P}(H < 3.2))^2$ $(\text{P}(H < 3.2))^2$ (M1)

$= 0.0111354575^2$

$= 0.0001239984137$

$= 0.000124$ (A1)

(d) (i) Let X be the number of short trees.

$\text{E}(X)$

$= (400)(0.0111354575)$ np (M1)

$= 4.454182999$

$= 4.45$ (A1)

(ii) The required probability

$= \text{P}(X > 8)$ $\text{P}(X > 8)$ (M1)

$= 1 - \text{P}(X \le 8)$ $1 - \text{P}(X \le 8)$ (M1)

$= 0.0372881747$

$= 0.0373$ (A1)

Exercise 4.10

(a) $\int_{\pi-1}^{\pi} k\,dx + \int_{\pi}^{2\pi} k\sin\left(\frac{1}{2}x\right)dx = 1$ Sum of probabilities (M1)

$k\left(\int_{\pi-1}^{\pi} dx + \int_{\pi}^{2\pi} \sin\left(\frac{1}{2}x\right)dx\right) = 1$

$3k = 1$

$k = \frac{1}{3}$ (A1)

(b) $P(X > 1.5\pi)$

$= \int_{1.5\pi}^{2\pi} \frac{1}{3}\sin\left(\frac{1}{2}x\right)dx$ $\int_{1.5\pi}^{2\pi} f(x)dx$ (M1)

$= 0.1952621459$

$= 0.195$ (A1)

(c) $E(X)$

$= \int_{\pi-1}^{\pi} x\cdot\frac{1}{3}dx + \int_{\pi}^{2\pi} x\cdot\frac{1}{3}\sin\left(\frac{1}{2}x\right)dx$ $\int_{a}^{b} x\cdot f(x)dx$ (A1)

$= 3.735987756$

$= 3.74$ (A1)

(d) $P(\pi - 1 < X < \pi)$

$= \int_{\pi-1}^{\pi} \frac{1}{3}dx$

$= \frac{1}{3}$

< 0.5 $\frac{1}{3} < 0.5$ (R1)

Thus, the median is between π and 2π.

$P(Q_2 < X < 2\pi) = 0.5$ $P(Q_2 < X < 2\pi) = 0.5$ (A1)

$\int_{Q_2}^{2\pi} \frac{1}{3}\sin\left(\frac{1}{2}x\right)dx = 0.5$

$\int_{Q_2}^{2\pi} \frac{1}{3}\sin\left(\frac{1}{2}x\right)dx - 0.5 = 0$

By considering the graph of

$y = \int_{Q_2}^{2\pi} \frac{1}{3}\sin\left(\frac{1}{2}x\right)dx - 0.5$, the horizontal

intercept is 3.6469532. GDC approach (M1)

Thus, the median of X is 3.65. (A1)

(e) The standard deviation of X

$= \sqrt{\text{Var}(X)}$ — $\sqrt{\text{Var}(X)}$ (M1)

$= \sqrt{\text{E}(X^2) - (\text{E}(X))^2}$ — $\text{E}(X^2) - (\text{E}(X))^2$ (M1)

$= \sqrt{\int_{\pi-1}^{\pi} x^2 \cdot \frac{1}{3} \text{d}x + \int_{\pi}^{2\pi} x^2 \cdot \frac{1}{3} \sin\left(\frac{1}{2}x\right) \text{d}x - (3.735987756)^2}$ — $\int_a^b x^2 \cdot f(x) \text{d}x$ (A1)

$= 1.002102044$

$= 1.00$ — (A1)