AI SL Practice Set 4 Paper 1 Solution

1. (a) (i) The distance travelled

$= 2\pi(1425000000)$ (M1) for valid approach

$= 8953539063\text{ km}$

$= 8950000000\text{ km}$ A1 N2

(ii) The distance travelled

$= \frac{2\pi(1425000000)}{(29)(365)}$ (M1) for valid approach

$= 845870.483\text{ km}$

$= 846000\text{ km}$ A1 N2

[4]

(b) $8.46 \times 10^5\text{ km}$ A2 N2

[2]

2. (a) $V = \frac{1}{3}\pi r^2 h$

$\therefore 128\pi = \frac{1}{3}\pi r^2(6)$ (A1) for correct equation

$r^2 = 64$

$r = 8$

Thus, the required radius is 8 cm. A1 N2

[2]

(b) l

$= \sqrt{r^2 + h^2}$ (M1) for valid approach

$= \sqrt{8^2 + 6^2}$

$= 10$

Thus, the required slant height is 10 cm. A1 N2

[2]

(c) The total surface area

$= \pi r^2 + \pi r l$

$= \pi(8)^2 + \pi(8)(10)$ (A1) for substitution

$= 144\pi\text{ cm}^2$ A1 N2

[2]

3. (a) (i) $-\frac{1}{26}$ A1 N1

(ii) -0.038462 A1 N1

[2]

(b) The percentage error

$$=\left|\frac{-0.039-(-0.038462)}{-0.038462}\right|\times 100\%$$ (A1) for substitution

$$=1.398783215\%$$

$$=1.40\%$$ A1 N2

[2]

4. (a) (i)
$$\begin{cases}7x+8y+5z=49\\4x+6y+10z=18\\11x+9y=82\end{cases}$$ A2 N2

(ii) $x=5$, $y=3$ and $z=-2$ A3 N3

[5]

(b) A team drops two points for losing a game. A1 N1

[1]

5. (a) (i) 20 hours A1 N1

(ii) 15 hours A1 N1

[2]

(b) 5 workers worked for more than 30 hours. Therefore, 12.5% of the workers worked for more than 30 hours. (R1) for correct argument

$\therefore k=30$ A1 N2

[2]

6. (a) (i) $\{0, 1, 2, 3, 4, 5\}$ A1 N1

(ii) $\{-1, 1, 11, 35, 79, 149\}$ A2 N2

[3]

(b) $g(x) = h(x)$

$x^3 + x^2 - 1 = 98\ln(0.57x)$

$x^3 + x^2 - 1 - 98\ln(0.57x) = 0$

By considering the graph of

$y = x^3 + x^2 - 1 - 98\ln(0.57x)$, $x = 1.9459391$ or

$x = 4.0546399$.

$\therefore x = 1.95$ or $x = 4.05$ A2 N2

[2]

7. (a) H_0: The outcomes follows the assigned distribution. A1 N1

[1]

(b) 50 A1 N1

[1]

(c) 4 A1 N1

[1]

(d) $p\text{-value} = 0.0003344965427$ (A1) for correct value

$p\text{-value} = 0.000334$ A1 N2

[2]

(e) The null hypothesis is rejected. A1

As $p\text{-value} < 0.05$. R1 N2

[2]

8.	(a)	(i)	c_n	A1	N1	
		(ii)	b_n	A1	N1	
						[2]
	(b)	(i)	1.25	A1	N1	
		(ii)	$\frac{3125}{128}$	A1	N1	
		(iii)	S_8			
			$=\frac{10(1.25^8-1)}{1.25-1}$	(A1) for substitution		
			$=198.4185791$			
			$=198$	A1	N2	
						[4]
9.	(a)	(i)	The radius			
			$=\sqrt{(10-6)^2+(12-14)^2}$	(A1) for substitution		
			$=4.472135955 \text{ km}$			
			$=4.47$ km	A1	N2	
		(ii)	4 km	A1	N1	
		(iii)	The apartment at P	A1	N1	
						[4]
	(b)		$x+y-20=0$	A2	N2	
						[2]

10. (a) The initial number of tigers. A1 N1

[1]

(b) 500 A1 N1

[1]

(c) The required number

$= P(7)$

$= \frac{500}{\ln 2}(\ln(7+2))$ (M1) for substitution

$= 1584.962501$

$= 1580$ A1 N2

[2]

(d) $P(t) = 1600$

$\therefore \frac{500}{\ln 2}(\ln(t+2)) = 1600$ (M1) for setting equation

$\frac{500}{\ln 2}(\ln(t+2)) - 1600 = 0$

By considering the graph of

$y = \frac{500}{\ln 2}(\ln(t+2)) - 1600$, $t = 7.1895868$.

Thus, the number of complete days needed is 8. A1 N2

[2]

11. (a) $\mathrm{E}(X) = 8.64$

$\therefore 0.72n = 8.64$ (A1) for correct equation

$n = 12$ A1 N2

[2]

(b) $\mathrm{Var}(X)$

$= (12)(0.72)(1-0.72)$ (A1) for substitution

$= 2.4192$ A1 N2

[2]

(c) $\mathrm{P}(X \geq 11)$

$= 1 - \mathrm{P}(X \leq 10)$ (A1) for substitution

$= 0.1099809898$

$= 0.110$ A1 N2

[2]

12. (a) By TVM Solver:

$N = 120$
$I\% = 4.5$
$PV = 0$
$PMT = -200$
$FV = ?$
$P/Y = 12$
$C/Y = 1$
$PMT : END$

(A2) for correct values

$FV = 30095.13482$

Thus, the value of the investment after ten years is $\$30100$. A1 N3

[3]

(b) By TVM Solver:

$N = 144$
$I\% = 4.5$
$PV = 0$
$PMT = ?$
$FV = 5 \times 30095.13482$
$P/Y = 12$
$C/Y = 1$
$PMT : END$

(A2) for correct values

$PMT = -794.6316652$

Thus, the new amount of deposit is $\$795$. A1 N3

[3]

13. (a) x

$= -\frac{b}{2a}$

$= -\frac{100}{2(-1)}$ (A1) for substitution

$= 50$ A1 N2

[2]

(b) The required maximum height

$= -50^2 + 100(50) - 1600$ A1

$= -2500 + 5000 - 1600$

$= 900 \text{ m}$ AG N0

[1]

(c) $V = 0$

$-x^2 + 100x - 1600 = 0$

$x = 20$ or $x = 80$ (A1) for correct values

The required horizontal distance

$= 80 - 20$ (M1) for valid approach

$= 60 \text{ m}$ A1 N3

[3]

14. (a) $P'(x) = 3x^2 - 135(2x) + 5400(1)$ (A1) for correct derivatives

$P'(x) = 3x^2 - 270x + 5400$ A1 N2

[2]

(b) $P'(x) = 0$

$3x^2 - 270x + 5400 = 0$ (M1) for setting equation

By considering the graph of

$y = 3x^2 - 270x + 5400$, $x = 30$ or

$x = 60$ (*Rejected*). (M1) for valid approach

Thus, the required number of loudspeakers is

30. A1 N3

[3]

(c) $67500 A1 N1

[1]