AA HL Practice Set 2 Paper 2 Solution

Section A

1.

$$\left(kx-\frac{4}{x}\right)^8=(kx)^8+\binom{8}{1}(kx)^7\left(-\frac{4}{x}\right)+\binom{8}{2}(kx)^6\left(-\frac{4}{x}\right)^2$$

$$+\binom{8}{3}(kx)^5\left(-\frac{4}{x}\right)^3+\binom{8}{4}(kx)^4\left(-\frac{4}{x}\right)^4+\ldots$$

(M1)(A1) for correct approach

$$\left(kx-\frac{4}{x}\right)^8=k^8x^8+8k^7x^7\left(-\frac{4}{x}\right)+28k^6x^6\left(\frac{16}{x^2}\right)$$

$$+56k^5x^5\left(-\frac{64}{x^3}\right)+70k^4x^4\left(\frac{256}{x^4}\right)+\ldots$$

(A1) for simplification

$$\left(kx-\frac{4}{x}\right)^8=k^8x^8-32k^7x^6+448k^6x^4$$

$$-3584k^5x^2+17920k^4+\ldots$$

A1

$$\therefore 448k^6:17920k^4=9:40$$

A1

$$\frac{448k^6}{17920k^4}=\frac{9}{40}$$

$$\frac{k^2}{40}=\frac{9}{40}$$

$k=-3$ or $k=3$ (*Rejected*) A1

[6]

2. (a) $A = 2\pi r^2 + 2\pi rh + 2\pi r^2$ (M2) for setting equation

$135\pi = 4\pi r^2 + 2\pi r(3.5)$ (A1) for substitution

$135 = 4r^2 + 7r$

$4r^2 + 7r - 135 = 0$ (M1) for quadratic equation

$(4r+27)(r-5) = 0$

$4r + 27 = 0$ or $r - 5 = 0$

$r = -\frac{27}{4}$ (*Rejected*) or $r = 5\text{ mm}$ A1

[5]

(b) The volume

$= \frac{4}{3}\pi r^3 + \pi r^2 h$ (M1) for valid approach

$= \frac{4}{3}\pi(5)^3 + \pi(5)^2(3.5)$

$= 798.4881328\text{ mm}^3$

$= 798\text{ mm}^3$ A1

[2]

3. (a) (i) $\cos A\hat{B}C = \frac{r^2 + (1.75r)^2 - (1.5r)^2}{2(r)(1.75r)}$ M1A1

$\cos A\hat{B}C = \frac{1.8125r^2}{3.5r^2}$ A1

$\cos A\hat{B}C = \frac{29}{56}$ AG

(ii) $A\hat{B}C = 1.026452178\text{ rad}$

$A\hat{B}C = 1.03\text{ rad}$ A1

[4]

(b) $\frac{1}{2}(BC)^2(A\hat{B}C) = 9.89$ (M1) for setting equation

$\frac{1}{2}r^2(\pi - 1.026452178) = 9.89$ (A1) for substitution

$r^2 = 9.35162474$

$r = 3.058042632$

$r = 3.06$ A1

[3]

4. $X \sim \mathrm{B}\left(5, \dfrac{2p}{p+2p+10}\right)$ (R1) for correct distribution

The standard deviation of X

$= \sqrt{5\left(\dfrac{2p}{3p+10}\right)\left(1-\dfrac{2p}{3p+10}\right)}$ (A1) for substitution

$= \sqrt{5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right)}$

$\therefore \sqrt{5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right)} > \dfrac{11}{10}$ (M1) for valid approach

$5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right) > \dfrac{121}{100}$ M1

$5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right) - \dfrac{121}{100} > 0$ A1

By considering the graph of

$y = 5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right) - \dfrac{121}{100}$,

$5.3435147 < p < 25.443002$.

Thus, the greatest value of p is 25. A1

[6]

5. $v = \int (8-8t)\mathrm{d}t$ (M1) for indefinite integral

$v = 8t - 8\left(\dfrac{1}{2}t^2\right) + C$ A1

$v = 8t - 4t^2 + C$

The initial velocity

$= 8(0) - 4(0)^2 + C$ (M1) for valid approach

$= C$

The difference between the velocities is $4\ \mathrm{ms}^{-1}$

$\therefore 8t - 4t^2 + C = C + 4$ or $\therefore 8t - 4t^2 + C = C - 4$ (A1) for correct approach

$4t^2 - 8t + 4 = 0$ or $4t^2 - 8t - 4 = 0$ A2

$4(t-1)^2 = 0$ or $t = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(-4)}}{2(4)}$

$t = 1$ or $t = 2.414213562$, $t = -0.4142135624$ (*Rejected*)

$\therefore m = 1$ or $m = 2.41$ A2

[8]

6. (a) By using row operations, the system

$$\left(\begin{array}{ccc|c} 2 & -1 & -3 & 3 \\ 1 & -4 & -6 & -17 \\ 3 & 1 & 2 & 21 \end{array}\right) \text{ is reduced to}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right).$$ (M1) for valid approach

Thus, the coordinates of P are $(5,4,1)$. A3

[4]

(b) $\mathbf{r} = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -4 \\ -6 \end{pmatrix}$ A2

[2]

7. (a) $\{x : -5 \le x \le 1\}$ A2

[2]

(b) $f(x) = 2-(x-1)^2$

$y = 2-(x-1)^2$

$\Rightarrow x = 2-(y-1)^2$ (M1) for swapping variables

$(y-1)^2 = 2-x$

$y-1 = \sqrt{2-x}$ (*Rejected*) or $y-1 = -\sqrt{2-x}$ A1

$y = -\sqrt{2-x}+1$

$\therefore f^{-1}(x) = -\sqrt{2-x}+1$ A1

[3]

(c) $(g^{-1} \circ f^{-1})(x) = \dfrac{x}{3}$

$f^{-1}(x) = g\left(\dfrac{x}{3}\right)$ M1

$g\left(\dfrac{x}{3}\right) = -\sqrt{2-x}+1$

$g\left(3\left(\dfrac{x}{3}\right)\right) = -\sqrt{2-3x}+1$ A1

$\therefore g(x) = -\sqrt{2-3x}+1$ A1

[3]

8. $\frac{2\pi}{B} = 2(4-0)$

$\frac{2\pi}{B} = 8$

$B = \frac{\pi}{4}$ A1

$5+\pi = A\sec\frac{\pi}{4}(0) + C$

$5+\pi = A+C$

$C = 5+\pi - A$

$5-\pi = A\sec\frac{\pi}{4}(4) + C$

$\therefore 5-\pi = A(-1)+5+\pi - A$ (M1) for substitution

$-2\pi = -2A$

$A = \pi$ A1

$C = 5+\pi-\pi$

$C = 5$ A1

[4]

9. (a) The total number of possible ways

$= \frac{14!}{14\times 2}$ (A2) for correct formula

$= 3113510400$ A1

[3]

(b) The number of possible ways

$= 3113510400 - \frac{2!\times 13!}{13\times 2}$ (A2) for correct formula

$= 2634508800$ A1

[3]

Section B

10. (a)

$\text{P}(L > 59.2) = 0.12$	(M1) for valid approach
$\text{P}\left(Z > \frac{59.2-\mu}{3.5}\right) = 0.12$	(A1) for correct approach
$\frac{59.2-\mu}{3.5} = 1.174986791$	A1
$59.2-\mu = 4.11245377$	
$\mu = 55.08754623$	
$\mu = 55.1$	A1
	[4]

(b)

$\text{P}(L < q) = 0.55$	
$\text{P}\left(Z < \frac{q-55.08754623}{3.5}\right) = 0.55$	(A1) for correct approach
$\frac{q-55.08754623}{3.5} = 0.1256613375$	
$q-55.08754623 = 0.4398146813$	
$q = 55.52736091$	A1
$\therefore q = 55.5$	A1
	[3]

(c) (i)

$X \sim \text{B}(10, 0.55)$	(R1) for correct distribution
$\text{E}(X) = (10)(0.55)$	(A1) for substitution
$\text{E}(X) = 5.5$	A1

(ii)

$\text{P}(X > 5) = 1-\text{P}(X \le 5)$	(M1) for valid approach
$\text{P}(X > 5) = 1-0.4955954083$	A1
$\text{P}(X > 5) = 0.5044045917$	
$\text{P}(X > 5) = 0.504$	A1
	[6]

(d)

$m\left(\frac{55\%}{55\%+33\%}\right)(0.8) + m\left(\frac{33\%}{55\%+33\%}\right)(1.1)$	(M1)(A1) for correct approach
$= (949)(1000)$	
$0.5m + 0.4125m = 949000$	A1
$0.9125m = 949000$	
$m = 1040000$	A1
	[4]

11. (a) When $0 \le t \le 1$,

$s(t) = \int \pi t \mathrm{d}t$ (M1) for valid approach

$s(t) = \frac{\pi}{2}t^2 + C$ (A1) for correct value

$s(0) = -\frac{\pi}{2}$

$\therefore \frac{\pi}{2}(0)^2 + C = -\frac{\pi}{2}$

$C = -\frac{\pi}{2}$

$s(1) = \frac{\pi}{2}(1)^2 - \frac{\pi}{2}$

$s(1) = 0$ (A1) for correct value

When $1 < t \le 5$,

$s(t) = \int \pi e^{1-t} \mathrm{d}t$ (M1) for valid approach

$s(t) = -\pi e^{1-t} + D$ (A1) for correct value

$s(1) = 0$

$\therefore -\pi e^{1-1} + D = 0$

$D = \pi$ (A1) for correct value

$s(5) = -\pi e^{1-5} + \pi$

$s(5) = -\pi e^{-4} + \pi$

$s(5) = \frac{\pi}{e^4}(e^4 - 1)$ (A1) for correct value

$$\therefore s(t) = \begin{cases} \frac{\pi}{2}t^2 - \frac{\pi}{2} & 0 \le t \le 1 \\ -\pi e^{1-t} + \pi & 1 < t \le 5 \\ \frac{\pi}{e^4}(e^4 - 1) & t > 5 \end{cases}$$ A1

[8]

(b) $a(t)=\begin{cases}\pi(1) & 0\le t\le 1\\ \pi e^{1-t}(-1) & 1<t\le 5\\ 0 & \text{otherwise}\end{cases}$ (M1) for valid approach

$a(t)=\begin{cases}\pi & 0\le t\le 1\\ -\pi e^{1-t} & 1<t\le 5\\ 0 & \text{otherwise}\end{cases}$ (A1) for correct values

$a(t)<-3$

$-\pi e^{1-t}<-3$ (M1) for setting inequality

$3-\pi e^{1-t}<0$

By considering the graph of $y=3-\pi e^{1-t}$,

$t<1.0461176$.

$\therefore 1<t<1.05$ A1

[4]

(c) (i) $\frac{\mathrm{d}s}{\mathrm{d}t}=\pi e^{1-t}$

$\frac{\mathrm{d}v}{\mathrm{d}t}=-\pi e^{1-t}$

$\therefore \frac{\mathrm{d}s}{\mathrm{d}v}$

$=\frac{\mathrm{d}s}{\mathrm{d}t}\div\frac{\mathrm{d}v}{\mathrm{d}t}$ M1

$=\frac{\pi e^{1-t}}{-\pi e^{1-t}}$ A1

$=-1$ AG

(ii) $\frac{\mathrm{d}t}{\mathrm{d}v}$

$=1\div\frac{\mathrm{d}v}{\mathrm{d}t}$ M1

$=\frac{1}{-\pi e^{1-t}}$ A1

$=-\frac{1}{\pi}e^{t-1}$ AG

[4]

12. (a) $|z|$

$= \left| \dfrac{\frac{4}{5}e^{i\theta}}{2} \right|$ (A1) for correct approach

$= \left| \dfrac{2}{5}e^{i\theta} \right|$

$= \dfrac{2}{5}\left| e^{i\theta} \right|$

$= \dfrac{2}{5}(1)$

$= \dfrac{2}{5}$ A1

[2]

(b) $\dfrac{2}{1-\frac{2}{5}e^{i\theta}}$ A2

[2]

(c) (i) $\dfrac{2}{1-\frac{2}{5}e^{i\theta}}$

$= \dfrac{10}{5-2e^{i\theta}}$

$= \dfrac{10(5-2e^{i(-\theta)})}{(5-2e^{i\theta})(5-2e^{i(-\theta)})}$ M1A1

$= \dfrac{50-20e^{i(-\theta)}}{25-10e^{i(-\theta)}-10e^{i\theta}+4}$ M1

$= \dfrac{50-20e^{i(-\theta)}}{29-10(e^{i(-\theta)}+e^{i\theta})}$

$= \dfrac{50-20(\cos(-\theta)+i\sin(-\theta))}{29-10(\cos(-\theta)+i\sin(-\theta)+\cos\theta+i\sin\theta)}$ A1

$= \dfrac{50-20(\cos\theta-i\sin\theta)}{29-10(\cos\theta-i\sin\theta+\cos\theta+i\sin\theta)}$

$= \dfrac{50-20\cos\theta+20i\sin\theta}{29-10(2\cos\theta)}$ M1

$= \dfrac{(50-20\cos\theta)+i(20\sin\theta)}{29-20\cos\theta}$ A1

$\frac{4}{5}\sin\theta+\frac{8}{25}\sin 2\theta+\frac{16}{125}\sin 3\theta+\cdots$

$=\frac{20\sin\theta}{29-20\cos\theta}$ M1A1

$\therefore \sin\theta+\frac{2}{5}\sin 2\theta+\frac{4}{25}\sin 3\theta+\cdots$

$=\frac{25\sin\theta}{29-20\cos\theta}$ AG

(ii) $2+\frac{4}{5}\cos\theta+\frac{8}{25}\cos 2\theta+\cdots=\frac{50-20\cos\theta}{29-20\cos\theta}$ M1A1

$\frac{4}{5}\cos\theta+\frac{8}{25}\cos 2\theta+\frac{16}{125}\cos 3\theta+\cdots$

$=\frac{50-20\cos\theta}{29-20\cos\theta}-2$ M1

$=\frac{50-20\cos\theta}{29-20\cos\theta}-\frac{2(29-20\cos\theta)}{29-20\cos\theta}$

$=\frac{50-20\cos\theta}{29-20\cos\theta}-\frac{58-40\cos\theta}{29-20\cos\theta}$ A1

$=\frac{50-20\cos\theta-58+40\cos\theta}{29-20\cos\theta}$ M1

$=\frac{-8+20\cos\theta}{29-20\cos\theta}$ A1

$\therefore \cos\theta+\frac{2}{5}\cos 2\theta+\frac{4}{25}\cos 3\theta+\cdots$

$=\frac{-10+25\cos\theta}{29-20\cos\theta}$ A1

$=\frac{5(-2+5\cos\theta)}{29-20\cos\theta}$ AG

[15]