# AA HL Practice Set 3 Paper 3 Solution

**1.** (a) $r_1 + r_2 = -\frac{a_1}{1}$ (A1) for substitution

$a_1 = -r_1 - r_2$ A1

$r_1 r_2 = \frac{a_0}{1}$ (A1) for substitution

$a_0 = r_1 r_2$ A1

[4]

(b) (i) $a_1$

$= -r_1 - r_2$

$= -(r_1 + r_2)$

$= -S_1$ A1

(ii) $\frac{S_1^2 - S_2}{2}$

$= \frac{(r_1 + r_2)^2 - (r_1^2 + r_2^2)}{2}$

$= \frac{r_1^2 + 2r_1 r_2 + r_2^2 - r_1^2 - r_2^2}{2}$ M1A1

$= \frac{2r_1 r_2}{2}$

$= r_1 r_2$ A1

$= a_0$

$\therefore a_0 = \frac{S_1^2 - S_2}{2}$ AG

[4]

(c) (i) $a_2 = -S_1$ A1

(ii) $\dfrac{S_1^{\ 2}-S_2}{2}$

$=\dfrac{(r_1+r_2+r_3)^2-(r_1^{\ 2}+r_2^{\ 2}+r_3^{\ 2})}{2}$

$=\dfrac{r_1^{\ 2}+r_1r_2+r_1r_3+r_1r_2+r_2^{\ 2}+r_2r_3+r_1r_3+r_2r_3+r_3^{\ 2}-r_1^{\ 2}-r_2^{\ 2}-r_3^{\ 2}}{2}$ M1A1

$=\dfrac{2r_1r_2+2r_1r_3+2r_2r_3}{2}$

$=r_1r_2+r_1r_3+r_2r_3$ R1

$=a_1$

$\therefore a_1=\dfrac{S_1^{\ 2}-S_2}{2}$ is true. A1

[5]

(d) $ka_0=S_1^{\ 3}-3S_1S_2+2S_3$

$k(-r_1r_2r_3)=(r_1+r_2+r_3)^3$
$-3(r_1+r_2+r_3)(r_1^{\ 2}+r_2^{\ 2}+r_3^{\ 2})+2(r_1^{\ 3}+r_2^{\ 3}+r_3^{\ 3})$ (A1) for correct approach

$-kr_1r_2r_3=(r_1+r_2+r_3)(r_1+r_2+r_3)^2$
$-3(r_1+r_2+r_3)(r_1^{\ 2}+r_2^{\ 2}+r_3^{\ 2})+2(r_1^{\ 3}+r_2^{\ 3}+r_3^{\ 3})$

$-kr_1r_2r_3$
$=(r_1+r_2+r_3)(r_1^{\ 2}+r_2^{\ 2}+r_3^{\ 2}+2r_1r_2+2r_1r_3+2r_2r_3)$ M1A1
$-3(r_1+r_2+r_3)(r_1^{\ 2}+r_2^{\ 2}+r_3^{\ 2})+2(r_1^{\ 3}+r_2^{\ 3}+r_3^{\ 3})$

The coefficient of $r_1r_2r_3$ on R.H.S.

$=2+2+2$ A1
$=6$

$\therefore k=-6$ A1

[5]

(e) $a_{n-1}=-S_1$ A1

$a_{n-2}=\dfrac{S_1^{\ 2}-S_2}{2}$ A1

$a_{n-3}=-\dfrac{1}{6}S_1^{\ 3}+\dfrac{1}{2}S_1S_2-\dfrac{1}{3}S_3$ A1

[3]

(f) $\begin{cases} u+v+w=14 \\ u^2+v^2+w^2=86 \\ u^3+v^3+w^3=560 \end{cases}$

Let $S_1=14$, $S_2=86$ and $S_3=560$. (M1) for valid approach

$u$, $v$ and $w$ are the roots of the equation

$x^3+a_2x^2+a_1x+a_0=0$, where $a_2=-S_1$,

$a_1=\frac{S_1{}^2-S_2}{2}$ and $a_0=-\frac{1}{6}S_1{}^3+\frac{1}{2}S_1S_2-\frac{1}{3}S_3$. A1

$a_2=-14$ A1

$a_1=\frac{14^2-86}{2}$

$a_1=55$ A1

$a_0=-\frac{1}{6}(14)^3+\frac{1}{2}(14)(86)-\frac{1}{3}(560)$

$a_0=-42$ A1

Therefore, $u$, $v$ and $w$ are the roots of the equation $x^3-14x^2+55x-42=0$. R1

By considering the graph of $y=x^3-14x^2+55x-42$, $x=1$, $x=6$ or $x=7$.

$\therefore u=1$, $v=6$, $w=7$ A3

[9]

**2.** (a)

$$\cos(A+B)x+\cos(A-B)x$$

$$\cos Ax\cos Bx-\sin Ax\sin Bx$$
$$+\cos Ax\cos Bx+\sin Ax\sin Bx$$ A2

$$=2\cos Ax\cos Bx$$ AG

[2]

(b)

$$\int_0^{\pi}\cos Ax\cos Bx\,\mathrm{d}x$$

$$=\frac{1}{2}\int_0^{\pi}(\cos(A+B)x+\cos(A-B)x)\mathrm{d}x$$ (A1) for substitution

$$=\frac{1}{2}\left[\frac{1}{A+B}\sin(A+B)x+\frac{1}{A-B}\sin(A-B)x\right]_0^{\pi}$$ A1

$$=\frac{1}{2}\left[\begin{array}{l}\left(\frac{1}{A+B}\sin(A+B)\pi+\frac{1}{A-B}\sin(A-B)\pi\right)\\-\left(\frac{1}{A+B}\sin 0+\frac{1}{A-B}\sin 0\right)\end{array}\right]$$ M1

$$=0$$ A1

[4]

(c) (i)

$$\frac{1}{z}$$

$$=\frac{1}{\cos\theta+\mathrm{i}\sin\theta}$$

$$=\frac{\cos\theta-\mathrm{i}\sin\theta}{(\cos\theta+\mathrm{i}\sin\theta)(\cos\theta-\mathrm{i}\sin\theta)}$$ M1

$$=\frac{\cos\theta-\mathrm{i}\sin\theta}{\cos^2\theta-\mathrm{i}^2\sin^2\theta}$$ A1

$$=\frac{\cos\theta-\mathrm{i}\sin\theta}{\cos^2\theta+\sin^2\theta}$$

$$=\cos\theta-\mathrm{i}\sin\theta$$ A1

(ii)

$$z+\frac{1}{z}=(\cos\theta+\mathrm{i}\sin\theta)+(\cos\theta-\mathrm{i}\sin\theta)$$ (M1) for valid approach

$$z+\frac{1}{z}=2\cos\theta$$

$$\cos\theta=\frac{1}{2}\left(z+\frac{1}{z}\right)$$ A1

[5]

(d) $\cos^3\theta$

$$=\left(\frac{1}{2}\right)^3\left(z+\frac{1}{z}\right)^3$$

$$=\frac{1}{8}\left(z^3+\binom{3}{1}z^2\cdot\frac{1}{z}+\binom{3}{2}z\cdot\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3\right)$$ M1A1

$$=\frac{1}{8}\left(\begin{array}{l}\cos 3\theta+\mathrm{i}\sin 3\theta+3\cos\theta+3\mathrm{i}\sin\theta\\+\frac{3}{\cos\theta+\mathrm{i}\sin\theta}+\frac{1}{\cos 3\theta+\mathrm{i}\sin 3\theta}\end{array}\right)$$ A1

$$=\frac{1}{8}\left(\begin{array}{l}\cos 3\theta+\mathrm{i}\sin 3\theta+3\cos\theta+3\mathrm{i}\sin\theta\\+3(\cos\theta-\mathrm{i}\sin\theta)+\cos 3\theta-\mathrm{i}\sin 3\theta\end{array}\right)$$ A1

$$=\frac{1}{8}(2\cos 3\theta+6\cos\theta)$$

$$=\frac{1}{4}(\cos 3\theta+3\cos\theta)$$ AG

[4]

(e) $\cos^n\theta$

$$=\left(\frac{1}{2}\right)^n\left(z+\frac{1}{z}\right)^n$$

$$=\frac{1}{2^n}\left(\begin{array}{l}z^n+\binom{n}{1}z^{n-1}\cdot\frac{1}{z}+\binom{n}{2}z^{n-2}\cdot\left(\frac{1}{z}\right)^2\\+\cdots+\binom{n}{n-1}z\cdot\left(\frac{1}{z}\right)^{n-1}+\left(\frac{1}{z}\right)^n\end{array}\right)$$ M1A1

$$=\frac{1}{2^n}\left(\begin{array}{l}z^n+\binom{n}{1}z^{n-2}+\binom{n}{2}z^{n-4}\\+\cdots+\binom{n}{n-1}\frac{1}{z^{n-2}}+\frac{1}{z^n}\end{array}\right)$$ M1

$$=\frac{1}{2^n}\sum_{r=0}^{n}\binom{n}{r}\cos(n-2r)\theta$$ A2

[5]

(f) $\int_0^{\pi} \cos 6x \cos^5 x \mathrm{d}x$

$$= \int_0^{\pi} \cos 6x \left( \frac{1}{2^5} \sum_{r=0}^{5} \binom{5}{r} \cos(5-2r)x \right) \mathrm{d}x$$ (A1) for substitution

$$= \frac{1}{32} \int_0^{\pi} \cos 6x \begin{pmatrix} \cos 5x + 5\cos 3x + 10\cos x \\ +10\cos(-x) + 5\cos(-3x) + \cos(-5x) \end{pmatrix} \mathrm{d}x$$ M1

$$= \frac{1}{32} \int_0^{\pi} \cos 6x \begin{pmatrix} \cos 5x + 5\cos 3x + 10\cos x \\ +10\cos x + 5\cos 3x + \cos 5x \end{pmatrix} \mathrm{d}x$$ A1

$$= \frac{1}{32} \int_0^{\pi} \cos 6x (2\cos 5x + 10\cos 3x + 20\cos x) \mathrm{d}x$$

$$= \frac{1}{32} \int_0^{\pi} 2\cos 6x \cos 5x \mathrm{d}x + \frac{1}{32} \int_0^{\pi} 10\cos 6x \cos 3x \mathrm{d}x + \frac{1}{32} \int_0^{\pi} 20\cos 6x \cos x \mathrm{d}x$$

$$= \frac{1}{16} \int_0^{\pi} \cos 6x \cos 5x \mathrm{d}x + \frac{5}{16} \int_0^{\pi} \cos 6x \cos 3x \mathrm{d}x + \frac{5}{8} \int_0^{\pi} \cos 6x \cos x \mathrm{d}x$$ A1

$$= \frac{1}{16}(0) + \frac{5}{16}(0) + \frac{5}{8}(0)$$

$$= 0$$ A1

[5]