Chapter 10 Solution

Exercise 28

1. (a) The coordinates of M

$=\left(\frac{-14+0}{2},\frac{-48+0}{2},\frac{0+0}{2}\right)$ (A1) for substitution

$=(-7,-24,0)$ A1 N2

[2]

(b) (i) $(-7,-24,n)$ A1 N1

(ii) $\sqrt{(-7-0)^2+(-24-0)^2+(n-0)^2}=65$ (M1) for setting equation

$\sqrt{625+n^2}=65$

$\sqrt{625+n^2}-65=0$

By considering the graph of

$\sqrt{625+n^2}-65=0$, $n=-60$ (*Rejected*) or

$n=60$. A1 N2

[3]

2. (a) The length of PQ

$=\sqrt{(45-15)^2+(85-25)^2+(15-35)^2}$ (A1) for substitution

$=70$ A1 N2

[2]

(b) The coordinates of M

$=\left(\frac{15+45}{2},\frac{25+85}{2},\frac{35+15}{2}\right)$ (A1) for substitution

$=(30,55,25)$ A1 N2

[2]

(c) (i) $(30,55,5)$ A1 N1

(ii) The length of QN

$=\sqrt{(45-30)^2+(85-55)^2+(15-5)^2}$ (A1) for substitution

$=35$ A1 N2

[3]

3.	(a)	The coordinates of M	
		$=\left(\frac{12+(-24)}{2}, \frac{-2+30}{2}, \frac{-8+4}{2}\right)$	(A1) for substitution
		$=(-6, 14, -2)$	A1 N2
			[2]
	(b)	The length of MN	
		$=\sqrt{40^2+9^2}$	(A1) for substitution
		$=41$	A1 N2
			[2]
	(c)	$\tan \text{P}\hat{\text{M}}\text{N} = \frac{30.75}{41}$	(M1) for tangent ratio
		$\text{P}\hat{\text{M}}\text{N} = 36.86989765^\circ$	
		Thus, the required angle of elevation is 36.9°.	A1 N2
			[2]
4.	(a)	$\sqrt{(29-(-16))^2+(-100-8)^2+(h-24)^2}=195$	(M1) for setting equation
		$\sqrt{13689+(h-24)^2}=195$	
		$\sqrt{13689+(h-24)^2}-195=0$	
		By considering the graph of	
		$y=\sqrt{13689+(h-24)^2}-195$, $h=-132$ or	
		$h=180$ (*Rejected*).	A1 N2
			[2]
	(b)	$\sin \text{Q}\hat{\text{P}}\text{R} = \frac{156}{195}$	(M1) for sine ratio
		$\text{Q}\hat{\text{P}}\text{R} = 53.13010235^\circ$	
		Thus, the required angle of depression is 53.1°.	A1 N2
			[2]
	(c)	$\left(\frac{29+x}{2}, \frac{-100+y}{2}, \frac{-132+z}{2}\right)=(-16, 8, 24)$	(M1) for valid approach
		$x=-61$, $y=116$ and $z=180$	
		Thus, the coordinates of S are $(-61, 116, 180)$.	A1 N2
			[2]

Exercise 29

1. (a) The gradient of L

$= \frac{11-6}{20-10}$ (M1) for valid approach

$= \frac{1}{2}$

The equation of L:

$y-11 = \frac{1}{2}(x-20)$ A1

$2y-22 = x-20$

$x-2y+2=0$ A1 N3

[3]

(b) The x-intercept of L is -2 A1

The y-intercept of L is 1 A1 N2

[2]

(c) $(-1, 0.5)$ A1 N1

[1]

2. (a) The gradient of L

$= \frac{-26-(-8)}{2-(-4)}$ (M1) for valid approach

$= -3$

The equation of L:

$y+8 = -3(x+4)$ A1

$y+8 = -3x-12$

$3x+y+20=0$ A1 N3

[3]

(b) The x-intercept of L is $-\frac{20}{3}$ A1

The y-intercept of L is -20 A1 N2

[2]

(c) $\frac{1}{3}$ A1 N1

[1]

3. (a) The gradient of L_1

$= \frac{37-1}{17-5}$ (M1) for valid approach

$= 3$

The equation of L_1:

$y-1=3(x-5)$ A1

$y-1=3x-15$

$3x-y-14=0$ A1 N3

[3]

(b) The gradient of L_2

$= -\frac{3}{-1}$ A1

$= 3$

As the gradients of L_1 and L_2 are the same,

L_1 and L_2 are parallel. R1 N2

[2]

4. (a) The gradient of L_1

$= \frac{40-0}{4-(-4)}$ (M1) for valid approach

$= 5$

The equation of L_1:

$y-0=5(x+4)$ A1

$y=5x+20$

$5x-y+20=0$ A1 N3

[3]

(b) The gradient of L_2

$= -\frac{1}{5}$ A1

The product of slopes

$= 5\times -\frac{1}{5}$

$= -1$

Thus, L_1 and L_2 are perpendicular. R1 N2

[2]

Exercise 30

1.	(a)	The gradient of L_1 is $\frac{1}{2}$	A1	
		The y-intercept of L_1 is 8	A1	N2
				[2]
	(b)	The gradient of L_2 is $\frac{1}{2}$	(A1) for correct value	
		The equation of L_2:		
		$y-5=\frac{1}{2}(x+2)$	A1	
		$2y-10=x+2$		
		$x-2y+12=0$	A1	N3
				[3]
2.	(a)	The gradient of L_1 is $-\frac{3}{2}$	A1	N1
				[1]
	(b)	$3(4)+2a-4=0$	(M1) for substitution	
		$2a+8=0$		
		$2a=-8$		
		$a=-4$	A1	N2
				[2]
	(c)	The gradient of L_2 is $-\frac{3}{2}$	(A1) for correct value	
		The equation of L_2:		
		$y+7=-\frac{3}{2}(x-1)$	A1	
		$2y+14=3-3x$		
		$3x+2y+11=0$	A1	N3
				[3]

3.	(a)	The gradient of L_1 is -3	A1		
		The x-intercept of L_1 is -7	A1	N2	
					[2]
	(b)	The gradient of L_2 is $\frac{1}{3}$	(A1) for correct value		
		The equation of L_2:			
		$y-0=\frac{1}{3}(x+7)$	A1		
		$3y=x+7$			
		$x-3y+7=0$	A1	N3	
					[3]

4.	(a)	(i) $\frac{1}{2}$	A1	N1	
		(ii) $-\frac{17}{4}$	A1	N1	
					[2]
	(b)	The gradient of L_2 is -2	(A1) for correct value		
		The equation of L_2:			
		$y+\frac{17}{4}=-2(x-0)$	A1		
		$4y+17=-8x$			
		$8x+4y+17=0$	A1	N3	
					[3]
	(c)	$8b+4(5.75)+17=0$	(M1) for substitution		
		$8b+40=0$			
		$8b=-40$			
		$b=-5$	A1	N2	
					[2]

Exercise 31

1. (a) The gradient of L_1

$= \frac{6-0}{-4-(-2)}$ (M1) for valid approach

$= -3$ A1 N2

[2]

(b) The equation of L_1:

$y-0=-3(x-(-2))$ (M1) for substitution

$y=-3x-6$ (A1) for simplification

$3x+y+6=0$ A1 N3

[3]

(c) The coordinates of C are $(2, 0)$. (A1) for correct values

The equation of L_2:

$y-0=-3(x-2)$ (M1) for substitution

$y=-3x+6$ A1 N3

[3]

(d) The coordinates of D

$= \left(\frac{-4+2}{2}, \frac{6+0}{2}\right)$ (A1) for substitution

$= (-1, 3)$ A1 N2

[2]

(e) The gradient of L_3

$= -1 \div -3$

$= \frac{1}{3}$ (A1) for correct value

The equation of L_3:

$y-3=\frac{1}{3}(x-(-1))$ (M1) for substitution

$3y-9=x+1$

$x-3y+10=0$ A1 N3

[3]

(f) $\text{CD} = \frac{k}{\sqrt{5}}\text{BD}$

$\sqrt{(-2-(-1))^2+(0-3)^2}$

$= \frac{k}{\sqrt{5}}\left(\sqrt{(2-(-1))^2+(0-3)^2}\right)$ (M1)(A1) for substitution

$\sqrt{18} = \frac{k}{\sqrt{5}}(\sqrt{10})$ (A1) for simplification

$k = 3$ A1 N4

[4]

2. (a) $\frac{k+30}{2}=25$ (M1) for setting equation

$k+30=50$

$k=20$ A1 N2

[2]

(b) The gradient of L_1

$=\frac{30-25}{40-20}$ (M1) for valid approach

$=\frac{1}{4}$ A1 N2

[2]

(c) The equation of L_1:

$y-20=\frac{1}{4}(x-0)$ (M1) for substitution

$y=\frac{1}{4}x+20$ A1 N2

[2]

(d) The gradient of L_2

$=-1\div\frac{1}{4}$

$=-4$ (A1) for correct value

The equation of L_2:

$y-20=-4(x-0)$ (M1) for substitution

$y=-4x+20$ A1 N3

[3]

(e) The equation of L_3:

$y-0=-4(x-20)$ (M1) for substitution

$y=-4x+80$ (A1) for simplification

$4x+y-80=0$ A1 N3

[3]

(f) $4r+r-80=0$ (M1) for substitution

$5r=80$

$r=16$ A1 N2

[2]

(g) $4x+\left(\frac{1}{4}x+20\right)-80=0$ (M1) for substitution

$\frac{17}{4}x=60$

$x=\frac{240}{17}$

$y=\frac{1}{4}\left(\frac{240}{17}\right)+20$ (M1) for substitution

$y=\frac{400}{17}$

Thus, the coordinates of D are $\left(\frac{240}{17},\frac{400}{17}\right)$. A1 N3

[3]

3. (a) The gradient of L_1

$= \frac{0-2k}{3k-0}$ (M1) for valid approach

$= \frac{-2k}{3k}$

$= -\frac{2}{3}$ A1 N2

[2]

(b) The equation of L_1:

$y-2k = -\frac{2}{3}(x-0)$ (M1) for substitution

$3y-6k = -2x$ (A1) for simplification

$2x+3y-6k = 0$ A1 N3

[3]

(c) $2(-30)+3(40)-6k = 0$ (M1) for substitution

$60 = 6k$

$k = 10$ A1 N2

[2]

(d) The gradient of L_2

$= -1 \div -\frac{2}{3}$

$= \frac{3}{2}$ (A1) for correct value

The equation of L_2:

$y-(-2.5) = \frac{3}{2}(x-15)$ (M1) for substitution

$2y+5 = 3x-45$

$3x-2y-50 = 0$ A1 N3

[3]

(e) $h = 15,\ k = -2.5$ A2 N2

[2]

(f) $20 = a(0-15)^2 - 2.5$ (M1) for setting equation

$22.5 = 225a$

$a = 0.1$ A1 N2

[2]

(g) $0.1(x-15)^2-2.5=0$ (M1) for setting equation

$(x-15)^2=25$

$x-15=\sqrt{25}$ or $x-15=-\sqrt{25}$

$x=20$ or $x=10$

Thus, the x-intercepts are 10 and 20. A2 N3

[3]

4. (a) $2x^2+4x-16=0$ (M1) for setting equation

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x=-4$ or $x=2$

$\therefore a=-4,\ b=2$ A2 N3

[3]

(b) -16 A1 N1

[1]

(c) $h=-1,\ k=-18$ A2 N2

[2]

(d) The gradient of VB

$=\dfrac{-18-0}{-1-2}$ (M1) for valid approach

$=6$ A1 N2

[2]

(e) The equation of VB:

$y-0=6(x-2)$ (M1) for substitution

$y=6x-12$ (A1) for simplification

$6x-y-12=0$ A1 N3

[3]

(f) (i) The gradient of CD

$=-1\div 6$

$=-\dfrac{1}{6}$ (A1) for correct value

$\dfrac{0-(-16)}{d-0}=-\dfrac{1}{6}$ (M1) for setting equation

$\dfrac{16}{d}=-\dfrac{1}{6}$

$d=-96$ A1 N3

(ii) $\dfrac{0-(-16)}{d-0}=6$ (M1) for setting equation

$\dfrac{16}{d}=6$

$d=\dfrac{8}{3}$ A1 N2

[5]

Exercise 32

1. (a) The gradient of L_1

$= \frac{0-100}{200-0}$ — (M1) for valid approach

$= -\frac{1}{2}$ — A1 N2

[2]

(b) The equation of L_1 :

$y-100 = -\frac{1}{2}(x-0)$ — (M1) for substitution

$2y-200 = -x$ — (A1) for simplification

$x+2y-200 = 0$ — A1 N3

[3]

(c) $m = 2$, $c = 0$ — A2 N2

[2]

(d) $x+2(2x)-200 = 0$ — (M1) for substitution

$5x = 200$

$x = 40$

$y = 2(40)$ — (M1) for substitution

$y = 80$

Thus, the coordinates of C are $(40, 80)$. — A1 N3

[3]

(e) The area of the triangle OAC

$= \frac{(100-0)(40-0)}{2}$ — (A1) for correct formula

$= 2000$ — A1 N2

[2]

(f) The area of the triangle OBC

$= \frac{(200-0)(80-0)}{2}$

$= 8000$ — (A1) for correct value

$\therefore \frac{8000}{2000} = \frac{r}{1}$ — (M1) for valid approach

$r = 4$ — A1 N3

[3]

2. (a) The gradient of L_1

$= \frac{0-20}{-60-(-20)}$ (M1) for valid approach

$= \frac{1}{2}$ A1 N2

[2]

(b) The equation of L_1:

$y-0=\frac{1}{2}(x-(-60))$ (M1) for substitution

$2y=x+60$ (A1) for simplification

$x-2y+60=0$ A1 N3

[3]

(c) The equation of L_2:

$y-0=\frac{1}{2}(x-(-30))$ (M1) for substitution

$y=\frac{1}{2}x+15$ A1 N2

[2]

(d) The gradient of L_3

$=-1\div\frac{1}{2}$

$=-2$ (A1) for correct value

The equation of L_3:

$y-20=-2(x-(-20))$ (M1) for substitution

$y=-2x-20$ A1 N3

[3]

(e) $\frac{1}{2}x+15=-2x-20$ (M1) for substitution

$\frac{5}{2}x=-35$

$x=-14$

$y=-2(-14)-20$ (M1) for substitution

$y=8$

Thus, the coordinates of D are $(-14, 8)$. A1 N3

[3]

(f)	$0 = -2x - 20$	(M1) for substitution
	$2x = -20$	
	$x = -10$	
	Thus, the coordinates of E are $(-10, 0)$.	(A1) for correct values
	The area of the triangle CDE	
	$= \dfrac{(-10-(-30))(8-0)}{2}$	(A1) for correct formula
	$= 80$	A1 N4

[4]

3. (a) $\frac{a-0}{0-30}=-\frac{4}{3}$ (M1) for setting equation

$-\frac{a}{30}=-\frac{4}{3}$

$a=40$ A1 N2

[2]

(b) The equation of L_1:

$y-0=-\frac{4}{3}(x-30)$ (M1) for substitution

$3y=-4x+120$ (A1) for simplification

$4x+3y-120=0$ A1 N3

[3]

(c) The gradient of L_2

$=-1\div-\frac{4}{3}$

$=\frac{3}{4}$ (A1) for correct value

The equation of L_2:

$y-0=\frac{3}{4}(x-30)$ (M1) for substitution

$y=\frac{3}{4}x-\frac{45}{2}$ A1 N3

[3]

(d) $c=\frac{3}{4}(3c)-\frac{45}{2}$ (M1) for substitution

$-\frac{5}{4}c=-\frac{45}{2}$

$c=18$ A1 N2

[2]

(e) $\mathrm{CE}^2=\mathrm{DE}^2-\mathrm{CD}^2$ (M1) for setting equation

$\mathrm{CE}^2=(15\sqrt{13})^2-30^2$ (A1) for substitution

$\mathrm{CE}^2=2025$

$\mathrm{CE}=45$ A1 N3

[3]

(f) The area of the triangle CDE

$=\frac{(45)(30)}{2}$ (M1) for valid approach

$=675$ A1 N2

[2]

4. (a) The gradient of L_1

$= \frac{10-0}{-20-0}$ (M1) for valid approach

$= -\frac{1}{2}$ A1 N2

[2]

(b) $p = 20$, $q = -10$ A2 N2

[2]

(c) The equation of L_1:

$y - 0 = -\frac{1}{2}(x-0)$ (M1) for substitution

$y = -\frac{1}{2}x$ A1 N2

[2]

(d) The gradient of L_2

$= -1 \div -\frac{1}{2}$

$= 2$ (A1) for correct value

The equation of L_2:

$y - 0 = 2(x-0)$ (M1) for substitution

$2x - y = 0$ A1 N3

[3]

(e) $\text{AB} = \sqrt{(-20-20)^2 + (10-(-10))^2}$ (M1) for valid approach

$\text{AB} = \sqrt{2000}$

$\frac{(\text{AB})(\text{OC})}{2} = 250$ (M1) for setting equation

$\frac{(\sqrt{2000})(\text{OC})}{2} = 250$

$\text{OC} = \sqrt{125}$ A1 N3

[3]

(f) Let $(c, 2c)$ be the coordinates of C.

$\sqrt{(c-0)^2 + (2c-0)^2} = \sqrt{125}$ (M1) for setting equation

$\sqrt{5c^2} = \sqrt{125}$

$5c^2 = 125$ (A1) for correct approach

$c^2 = 25$

$c = -5$ (*Rejected*) or $c = 5$ (A1) for correct value

Thus, the coordinates of C are $(5, 10)$. A1 N4

[4]