Exercise 3.1

(a) The coordinates of the mid-point

$= \left(\frac{0+7}{2}, \frac{0+24}{2}, \frac{0+0}{2}\right)$ $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$ (M1)

$= (3.5, 12, 0)$ (A1)

(b) (i) $(7, 24, h)$ (A1)

(ii) $\sqrt{(7-0)^2+(24-0)^2+(h-0)^2} = \sqrt{1025}$ Correct equation (A1)

$49+576+h^2 = 1025$

$h^2 = 400$

$h = 20$ (A1)

Exercise 3.2

(a) (i) $\sin^2 A+\cos^2 A=1$ — $\sin^2 A+\cos^2 A\equiv 1$ (M1)

$\therefore \sin^2 A+\left(-\frac{1}{2}\right)^2=1$ — $\cos A=-\frac{1}{2}$ (A1)

$\sin^2 A=\frac{3}{4}$

$\sin A=\sqrt{\frac{3}{4}}$ (*Rejected*) or $\sin A=-\sqrt{\frac{3}{4}}$

$\sin A=-\frac{\sqrt{3}}{2}$ — (A1)

(ii) $\tan A$

$=\frac{\sin A}{\cos A}$ — $\tan A\equiv\frac{\sin A}{\cos A}$ (M1)

$=-\frac{\sqrt{3}}{2}\div-\frac{1}{2}$ — $-\frac{\sqrt{3}}{2}$ & $-\frac{1}{2}$ (A1)

$=\sqrt{3}$ — (A1)

(iii) $\sin 2A$

$=2\sin A\cos A$ — $\sin 2A\equiv 2\sin A\cos A$ (M1)

$=2\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)$ — $-\frac{\sqrt{3}}{2}$ & $-\frac{1}{2}$ (A1)

$=\frac{\sqrt{3}}{2}$ — (A1)

(b) (i) $\cos 2B$

$=1-2\sin^2 B$ — $\cos 2B\equiv 1-2\sin^2 B$ (M1)

$=1-2\left(-\frac{1}{3}\right)^2$ — $\sin B=-\frac{1}{3}$ (A1)

$=\frac{7}{9}$ — (A1)

(ii) $\sin^2 2B + \cos^2 2B = 1$ — $\sin^2 2B + \cos^2 2B \equiv 1$ (M1)

$\therefore \sin^2 2B + \left(\frac{7}{9}\right)^2 = 1$ — $\cos 2B = \frac{7}{9}$ (A1)

$\sin^2 2B = \frac{32}{81}$

$\sin 2B = \sqrt{\frac{32}{81}}$ (*Rejected*) or

$\sin 2B = -\sqrt{\frac{32}{81}}$

$\sin 2B = -\frac{\sqrt{32}}{9}$ (A1)

(c) (i) $\tan 2A$

$= \frac{2\tan A}{1-\tan^2 A}$ — $\tan 2A \equiv \frac{2\tan A}{1-\tan^2 A}$ (M1)

$= \frac{2\sqrt{3}}{1-(\sqrt{3})^2}$ — $\tan A = \sqrt{3}$ (A1)

$= \frac{2\sqrt{3}}{-2}$ — $1-(\sqrt{3})^2 = -2$ (A1)

$= -\sqrt{3}$ (AG)

(ii) $\sin(A-B)$

$= \sin A\cos B - \cos A\sin B$ — $\sin A\cos B - \cos A\sin B$ (M1)

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{8}}{3}\right) - \left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)$ — $-\frac{\sqrt{3}}{2}$, $\frac{\sqrt{8}}{3}$, $-\frac{1}{2}$ & $-\frac{1}{3}$ (A1)

$= \frac{-\sqrt{24}-1}{6}$

$= -\frac{\sqrt{24}+1}{6}$ (AG)

(iii) $\cos(A-B)$

$=\cos A\cos B+\sin A\sin B$ $\qquad$ $\cos A\cos B+\sin A\sin B$ (M1)

$=\left(-\frac{1}{2}\right)\left(\frac{\sqrt{8}}{3}\right)+\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{3}\right)$ $\qquad$ $-\frac{1}{2}$, $\frac{\sqrt{8}}{3}$, $-\frac{\sqrt{3}}{2}$ & $-\frac{1}{3}$ (A1)

$=\frac{-\sqrt{8}+\sqrt{3}}{6}$ $\qquad$ (AG)

Exercise 3.3

(a) $0 < x < \frac{2\pi}{3} \Rightarrow 0 < 3x < 2\pi$ — $0 < 3x < 2\pi$ (M1)

$\cos 3x = -\frac{1}{2}$

$\because \cos\left(\pi - \frac{\pi}{3}\right)$ and $\cos\left(\pi + \frac{\pi}{3}\right) = -\frac{1}{2}$ — 2 possible cases (M1)

$\therefore 3x = \pi - \frac{\pi}{3}$ or $\pi + \frac{\pi}{3}$ — 2 possible angles (A1)

$3x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

$x = \frac{2\pi}{9}$ or $\frac{4\pi}{9}$ — (A1)(A1)

(b) $2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$ — $(2\sin x - 1)(\sin x - 1)$ (M1)

$2\sin x - 1 = 0$ or $\sin x - 1 = 0$

$\sin x = \frac{1}{2}$ or $\sin x = 1$ — $\frac{1}{2}$ & 1 (A1)

$\because \sin\left(\pi - \frac{\pi}{6}\right) = \frac{1}{2}$ and $\sin x \neq 1$ for $\frac{\pi}{2} < x < \pi$ — 1 possible case (M1)

$\therefore x = \frac{5\pi}{6}$ — (A1)

(c) $-\frac{\pi}{2} \le x \le \frac{\pi}{2} \Rightarrow -\pi \le 2x \le \pi$ — $-\pi \le 2x \le \pi$ (M1)

$\sec^2 2x = 1$

$\therefore \cos^2 2x = 1$

$\cos 2x = \sqrt{1}$ or $\cos 2x = -\sqrt{1}$

$\cos 2x = 1$ or $\cos 2x = -1$

$\because \cos 0 = 1$, $\cos(-\pi)$ and $\cos \pi = -1$ — 3 possible cases (M1)

$\therefore 2x = 0$, $-\pi$ or π — 3 possible angles (A1)

$x = -\frac{\pi}{2}$, 0 or $\frac{\pi}{2}$ — (A1)(A1)(A1)

Exercise 3.4

(a) (i) 8 m (A1)

(ii) 62 m (A1)

(b) (i) a

$= \dfrac{62-8}{2}$ $\dfrac{y_{\max} - y_{\min}}{2}$ (M1)

$= 27$ (A1)

(ii) d

$= \dfrac{62+8}{2}$ $\dfrac{y_{\max} + y_{\min}}{2}$ (M1)

$= 35$ (A1)

(iii) b

$= \dfrac{2\pi}{\text{Period}}$ $\dfrac{2\pi}{\text{Period}}$ (M1)

$= \dfrac{2\pi}{\frac{60}{6}}$

$= \dfrac{\pi}{5}$ (A1)

(c) $y = 27\sin\left(\dfrac{\pi}{5}(t-c)\right) + 35$

$\therefore 8 = 27\sin\left(\dfrac{\pi}{5}(0-c)\right) + 35$ $t = 0$ & $y = 8$ (M1)

$-27 = 27\sin\left(-\dfrac{\pi}{5}c\right)$

$\sin\left(-\dfrac{\pi}{5}c\right) = -1$

$-\dfrac{\pi}{5}c = -\dfrac{\pi}{2}$ $\sin\left(-\dfrac{\pi}{2}\right) = -1$ (A1)

$C = 2.5$ (AG)

(d) $27\sin\left(\frac{\pi}{5}(t-2.5)\right)+35\geq 50$ Correct inequality (A1)

$27\sin\left(\frac{\pi}{5}(t-2.5)\right)-15\geq 0$

By considering the graph of

$y=27\sin\left(\frac{\pi}{5}(t-2.5)\right)-15$, the graph is above

The horizontal axis when

$3.4374719<x<6.5625281$,

$13.437472<x<16.562528$

or $23.437472<x<26.562528$. GDC approach (M1)

The amount of time

$=3(6.5625281-3.4374719)$ $3(t_2-t_1)$ (A1)

$=9.3751686$ min

$=$ 9.38 min (A1)

Exercise 3.5

(a) (i) $\overset{\frown}{\mathrm{ACB}}$

$= (8)\left(\dfrac{3\pi}{4}\right)$ — $s = r\theta$ (A1)

$= 6\pi$ m — (A1)

(ii) The exact perimeter

$= 6\pi + 8 + 8$ — Sum of three sides (M1)

$= (6\pi + 16)$ m — (A1)

(b) (i) The area of OACB

$= \dfrac{1}{2}(8)^2\left(\dfrac{3\pi}{4}\right)$ — $A = \dfrac{1}{2}r^2\theta$ (A1)

$= 24\pi$ m² — (A1)

(ii) The area of OAB

$= \dfrac{1}{2}(8)(8)\sin\dfrac{3\pi}{4}$ — $A = \dfrac{1}{2}ab\sin \mathrm{C}$ (A1)

$= 32\left(\dfrac{\sqrt{2}}{2}\right)$ — $\sin\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$ (A1)

$= 16\sqrt{2}$ m² — (A1)

(iii) The area of ACB

$= 24\pi - 16\sqrt{2}$ — Difference of areas (M1)

$= 8(3\pi - 2\sqrt{2})$ m² — (A1)

Exercise 3.6

$\mathrm{QR}^2 = \mathrm{PQ}^2 + \mathrm{PR}^2 - 2(\mathrm{PQ})(\mathrm{PR})\cos \mathrm{Q\hat{P}R}$ — Cosine rule (M1)

$12^2 = 19^2 + \mathrm{PR}^2 - 2(19)(\mathrm{PR})\cos 0.483$ — 12, 19 & 0.483 (A1)

$\mathrm{PR}^2 - (38\cos 0.483)\mathrm{PR} + 217 = 0$

$\mathrm{PR} = 8.694390054$ or $\mathrm{PR} = 24.95862259$ — Two values (A1)

The required difference

$= 24.95862259 - 8.694390054$

$= 16.26423254$

$= 16.3$ — (A1)

Exercise 3.7

(a) $\dfrac{\text{AC}}{\sin \text{A}\hat{\text{B}}\text{C}} = \dfrac{\text{AB}}{\sin \text{A}\hat{\text{C}}\text{B}}$ Sine rule (M1)

$\dfrac{\text{AC}}{\sin\left((360^\circ - 310^\circ)\times \dfrac{\pi}{180^\circ}\right)} = \dfrac{60}{\sin 1.45}$ $50^\circ \times \dfrac{\pi}{180^\circ}$, 60 & 1.45 (A1)

$\dfrac{\text{AC}}{\sin \dfrac{5\pi}{18}} = \dfrac{60}{\sin 1.45}$

$\text{AC} = 46.30005551$ km

$\text{AC} = 46.3$ km (A1)

(b) The area of ABC

$= \dfrac{1}{2}(\text{AB})(\text{AC})\sin \text{B}\hat{\text{A}}\text{C}$ $A = \dfrac{1}{2}ab\sin \text{C}$ (M1)

$= \dfrac{1}{2}(60)(46.30005551)\sin\left(\pi - 1.45 - \dfrac{5\pi}{18}\right)$ 60, 46.30005551 & $\pi - 1.45 - \dfrac{5\pi}{18}$ (A1)

$= 1014.546384$ km^2

$= 1010$ km^2 (A1)

(c) The area of ACD

$= (1014.546384)(1.44)$

$= 1460.946793$ 1460.946793 (A1)

$\dfrac{1}{2}(\text{AC})(\text{CD})\sin \text{A}\hat{\text{C}}\text{D} = 1460.946793$ $A = \dfrac{1}{2}ab\sin \text{C}$ (M1)

$\dfrac{1}{2}(46.30005551)(75)\sin\theta = 1460.946793$ 46.30005551 & 75 (A1)

$\sin\theta = 0.8414370289$

$\theta = 0.999937157$ radian

$\theta = 1.00$ radian (A1)

(d) $\frac{\mathrm{BC}}{\sin \mathrm{B\hat{A}C}} = \frac{\mathrm{AB}}{\sin \mathrm{A\hat{C}B}}$ Sine rule (M1)

$\frac{\mathrm{BC}}{\sin\left(\pi - 1.45 - \frac{5\pi}{18}\right)} = \frac{60}{\sin 1.45}$ $\pi - 1.45 - \frac{5\pi}{18}$, 60 & 1.45 (A1)

$\mathrm{BC} = 44.14654149$

$\mathrm{BD}^2 = \mathrm{CD}^2 + \mathrm{BC}^2 - 2(\mathrm{CD})(\mathrm{BC})\cos \mathrm{B\hat{C}D}$ Cosine rule (M1)

$\mathrm{BD}^2 = 75^2 + 44.14654149^2$ $75, 44.14654149$

$-2(75)(44.14654149)\cos(1.45 + 0.999937157)$ & $1.45 + 0.999937157$ (A1)

$\mathrm{BD} = 112.5793436$

$\frac{\mathrm{BD}}{\text{Speed of Car Q}} = \frac{\mathrm{BC} + \mathrm{CD}}{60}$ Same time (M1)

$\frac{112.5793436}{\text{Speed of Car Q}} = \frac{44.14654149 + 75}{60}$

$\text{Speed of Q} = 56.69288031$

Thus, the speed of Car Q is 56.7 km/h. (A1)

Exercise 3.8

(a) The volume

$= \frac{2}{3}\pi r^3$ $\quad V = \frac{2}{3}\pi r^3$ (M1)

$= \frac{2}{3}\pi (20)^3$ $\quad r = 20$ (A1)

$= 16755.16082$ cm³

$= 16800$ cm³ (A1)

(b) The total surface area

$= 3\pi r^2$ $\quad A = 3\pi r^2$ (M1)

$= 3\pi (20)^2$ $\quad r = 20$ (A1)

$= 3769.911184$ cm²

$= 3770$ cm² (A1)

(c) $V = \frac{1}{3}Ah$ $\quad V = \frac{1}{3}Ah$ (M1)

$(16755.16082)(4) = \frac{1}{3}(\text{AD})^2(47)$ $\quad V = 67020.64328$ & $h = 47$ (A1)

$\text{AD}^2 = 4277.913401$

$\text{AD} = 65.40575969$ cm

$\text{AD} = 65.4$ cm (A1)

(d) $\tan \text{O}\hat{\text{M}}\text{V} = \frac{\text{OV}}{\text{OM}}$ $\quad$ Tangent ratio (M1)

$\tan \text{O}\hat{\text{M}}\text{V} = \frac{47}{65.40575969 \div 2}$

$\text{O}\hat{\text{M}}\text{V} = 0.9628907079$ radians

Thus, the required angle is 0.963 radians. (A1)

(e) OD

$= \sqrt{\mathrm{OM}^2 + \mathrm{DM}^2}$

$= \sqrt{\left(\frac{65.40575969}{2}\right)^2 + \left(\frac{65.40575969}{2}\right)^2}$

$= 46.24885621$ 46.24885621 (A1)

$\tan \mathrm{O\hat{D}V} = \frac{\mathrm{OV}}{\mathrm{OD}}$ Tangent ratio (M1)

$\tan \mathrm{O\hat{D}V} = \frac{47}{46.24885621}$

$\mathrm{O\hat{D}V} = 0.7934532493$ radians

Thus, the required angle is 0.793 radians. (A1)

Exercise 3.9

(a) $|\mathbf{r}|^2$

$= |\mathbf{p} + \lambda\mathbf{q}|^2$

$= (\mathbf{p} + \lambda\mathbf{q}) \cdot (\mathbf{p} + \lambda\mathbf{q})$ $\quad |\mathbf{u}|^2 = \mathbf{u} \cdot \mathbf{u}$ (M1)

$= \mathbf{p} \cdot \mathbf{p} + \mathbf{p} \cdot \lambda\mathbf{q} + \lambda\mathbf{q} \cdot \mathbf{p} + \lambda\mathbf{q} \cdot \lambda\mathbf{q}$ $\quad$ Expansion (A1)

$= \mathbf{p} \cdot \mathbf{p} + \lambda\mathbf{p} \cdot \mathbf{q} + \lambda\mathbf{q} \cdot \mathbf{p} + \lambda^2\mathbf{q} \cdot \mathbf{q}$

$= \mathbf{p} \cdot \mathbf{p} + \lambda\mathbf{p} \cdot \mathbf{q} + \lambda\mathbf{p} \cdot \mathbf{q} + \lambda^2\mathbf{q} \cdot \mathbf{q}$ $\quad \mathbf{p} \cdot \mathbf{q} = \mathbf{q} \cdot \mathbf{p}$ (M1)

$= |\mathbf{p}|^2 + 2\lambda\mathbf{p} \cdot \mathbf{q} + \lambda^2|\mathbf{q}|^2$ $\quad$ (AG)

(b) $(\mathbf{p} \times (\mathbf{q} + \mathbf{r})) \cdot (\mathbf{p} + \mathbf{q} + \mathbf{r})$

$= (\mathbf{p} \times \mathbf{q} + \mathbf{p} \times \mathbf{r}) \cdot (\mathbf{p} + \mathbf{q} + \mathbf{r})$ $\quad \mathbf{p} \times (\mathbf{q} + \mathbf{r}) = \mathbf{p} \times \mathbf{q} + \mathbf{p} \times \mathbf{r}$ (M1)

$= (\mathbf{p} \times \mathbf{q}) \cdot \mathbf{p} + (\mathbf{p} \times \mathbf{q}) \cdot \mathbf{q} + (\mathbf{p} \times \mathbf{q}) \cdot \mathbf{r}$

$+ (\mathbf{p} \times \mathbf{r}) \cdot \mathbf{p} + (\mathbf{p} \times \mathbf{r}) \cdot \mathbf{q} + (\mathbf{p} \times \mathbf{r}) \cdot \mathbf{r}$ $\quad$ Expansion (M1)

$= \mathbf{0} + \mathbf{0} + (\mathbf{p} \times \mathbf{q}) \cdot \mathbf{r} + \mathbf{0} + (\mathbf{p} \times \mathbf{r}) \cdot \mathbf{q} + \mathbf{0}$ $\quad (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = \mathbf{0}$ (A1)

$= \mathbf{q} \cdot (\mathbf{r} \times \mathbf{p}) + (\mathbf{p} \times \mathbf{r}) \cdot \mathbf{q}$

$= \mathbf{q} \cdot (\mathbf{r} \times \mathbf{p}) - (\mathbf{r} \times \mathbf{p}) \cdot \mathbf{q}$ $\quad \mathbf{p} \times \mathbf{r} = -(\mathbf{r} \times \mathbf{p})$ (A1)

$= 0$ $\quad$ (AG)

Exercise 3.10

(a) (i) $\mathbf{b}_1 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ (A1)

(ii) $\mathbf{r}_1 = \mathbf{a}_1 + t\mathbf{b}_1$ $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ (M1)

$\mathbf{r}_1 = \begin{pmatrix} 10 \\ 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ (A1)

(b) $\mathbf{r}_1 = \begin{pmatrix} 10 \\ 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

$\begin{cases} x = 10 + 2t \\ y = 3t \\ z = -5 + t \end{cases}$ Parametric form (M1)

$\begin{cases} x - 10 = 2t \\ y = 3t \\ z + 5 = t \end{cases}$

$\therefore \dfrac{x-10}{2} = \dfrac{y}{3} = z + 5$ (A1)

(c) $\dfrac{x-10}{2} = \dfrac{y}{3} = z + 5$

$\because \dfrac{0-10}{2} = \dfrac{-15}{3} = -10 + 5 = -5$ $x = 0$, $y = -15$ & $z = -10$ (M1)

Thus, the point $(0, -15, -10)$ lies on L_1. (AG)

(d) $\mathbf{r}_1 = \begin{pmatrix} 10 \\ 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ & $\mathbf{r}_2 = \begin{pmatrix} 12 \\ 9 \\ -5 \end{pmatrix} + s\begin{pmatrix} -8 \\ 0 \\ -6 \end{pmatrix}$

$\therefore \begin{cases} x = 10 + 2t \\ y = 3t \\ z = -5 + t \end{cases}$ & $\begin{cases} x = 12 - 8s \\ y = 9 \\ z = -5 - 6s \end{cases}$

$3t = 9$ Equating y (M1)

$t = 3$

$10 + 2(3) = 12 - 8s$ Equating x (M1)

$4 = -8s$

$s = -0.5$ $t = 3$ & $s = -0.5$ (A1)

$\because -5 + 3 = -5 - 6(-0.5) = -2$ Checking z (R1)

Thus, the point of intersection exists.

$\begin{cases} x = 12 - 8(-0.5) = 16 \\ y = 9 \\ z = -5 - 6(-0.5) = -2 \end{cases}$

Thus, the coordinates of P are $(16, 9, -2)$. (A1)

(e) $\mathbf{b}_1 \cdot \mathbf{b}_2 = |\mathbf{b}_1||\mathbf{b}_2|\cos\theta$. Scalar product (M1)

$\therefore \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -8 \\ 0 \\ -6 \end{pmatrix} = \left|\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}\right| \left|\begin{pmatrix} -8 \\ 0 \\ -6 \end{pmatrix}\right| \cos\theta$

$(2)(-8) + (3)(0) + (1)(-6)$

$= (\sqrt{2^2 + 3^2 + 1^2})(\sqrt{(-8)^2 + 0^2 + (-6)^2})\cos\theta$ Substitution (A1)

$\cos\theta = \dfrac{-22}{\sqrt{14} \cdot \sqrt{100}}$ $\dfrac{-22}{\sqrt{14} \cdot \sqrt{100}}$ (A1)

$\theta = 2.199349088$ radians

$\therefore$ The required obtuse angle is 2.20 radians. (A1)

(f) (i) $\mathbf{n}$

$$=\begin{pmatrix}4\\0\\3\end{pmatrix}\times\begin{pmatrix}1\\1\\1\end{pmatrix}$$ $\begin{pmatrix}4\\0\\3\end{pmatrix}\times\begin{pmatrix}1\\1\\1\end{pmatrix}$ (M1)

$$=\begin{pmatrix}-3\\-1\\4\end{pmatrix}$$ (A1)

(ii) $\mathbf{r}\cdot\mathbf{n}=\mathbf{a}\cdot\mathbf{n}$ $\mathbf{r}\cdot\mathbf{n}=\mathbf{a}\cdot\mathbf{n}$ (M1)

$$\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}-3\\-1\\4\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}\cdot\begin{pmatrix}-3\\-1\\4\end{pmatrix}$$ $\begin{pmatrix}-3\\-1\\4\end{pmatrix}$ & $\begin{pmatrix}0\\0\\0\end{pmatrix}$ (A1)

$$-3x-y+4z=(0)(-3)+(0)(-1)+(0)(4)$$

$$3x+y-4z=0$$ (A1)

(g) Let α be the angle between $\mathbf{b}_1$ and $\mathbf{n}$.

$$\mathbf{b}_1\cdot\mathbf{n}=|\mathbf{b}_1||\mathbf{n}|\cos\alpha$$ Scalar product (M1)

$$\therefore\begin{pmatrix}2\\3\\1\end{pmatrix}\cdot\begin{pmatrix}-3\\-1\\4\end{pmatrix}=\left|\begin{pmatrix}2\\3\\1\end{pmatrix}\right|\left|\begin{pmatrix}-3\\-1\\4\end{pmatrix}\right|\cos\alpha$$

$$(2)(-3)+(3)(-1)+(1)(4)$$

$$=(\sqrt{2^2+3^2+1^2})(\sqrt{(-3)^2+(-1)^2+4^2})\cos\alpha$$ Substitution (A1)

$$\cos\alpha=\frac{-5}{\sqrt{14}\cdot\sqrt{26}}$$ $\frac{-5}{\sqrt{14}\cdot\sqrt{26}}$ (A1)

$\alpha=1.835964129$ radians

The required acute angle

$$=1.835964129-\frac{\pi}{2}$$ $\alpha-\frac{\pi}{2}$ (M1)

$=0.2651678023$ radians

$=0.265$ radians (A1)

(h) $L_1: \begin{cases} x = 10 + 2t \\ y = 3t \\ z = -5 + t \end{cases}$ and $\Pi: 3x + y - 4z = 0$

$\therefore 3(10 + 2t) + 3t - 4(-5 + t) = 0$ $10 + 2t$, $3t$ & $-5 + t$ (A1)

$30 + 6t + 3t + 20 - 4t = 0$

$5t = -50$

$t = -10$ -10 (A1)

$\begin{cases} x = 10 + 2(-10) = -10 \\ y = 3(-10) = -30 \\ z = -5 + (-10) = -15 \end{cases}$

$\therefore$ The coordinates of Q are $(-10, -30, -15)$. (A1)

Exercise 3.11

(a) By solving the system $\begin{cases} x-7y=-1 \\ x-y+2z=0 \end{cases}$,

$x=\frac{1}{6}-\frac{7}{3}z$ and $y=\frac{1}{6}-\frac{1}{3}z$. GDC approach (M1)

Let $z=t$.

$\therefore \begin{cases} x=\frac{1}{6}-\frac{7}{3}t \\ y=\frac{1}{6}-\frac{1}{3}t \\ z=t \end{cases}$ Parametric form (A1)

$\mathbf{r}=\begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ 0 \end{pmatrix}+t\begin{pmatrix} -\frac{7}{3} \\ -\frac{1}{3} \\ 1 \end{pmatrix}$ (A1)

(b) $\mathbf{n}_1\cdot\mathbf{n}_3=|\mathbf{n}_1||\mathbf{n}_3|\cos\theta$. Scalar product (M1)

$\therefore \begin{pmatrix} 1 \\ -7 \\ 0 \end{pmatrix}\cdot\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}=\left|\begin{pmatrix} 1 \\ -7 \\ 0 \end{pmatrix}\right|\left|\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}\right|\cos\theta$

$(1)(2)+(-7)(1)+(0)(5)$

$=(\sqrt{1^2+(-7)^2+0^2})(\sqrt{2^2+1^2+5^2})\cos\theta$ Substitution (A1)

$\cos\theta=\frac{-5}{\sqrt{50}\cdot\sqrt{30}}$ $\frac{-5}{\sqrt{50}\cdot\sqrt{30}}$ (A1)

$\theta=1.700257098$ radians

$\pi-1.700257098=1.441335556$

$\therefore$ The required acute angle is 1.44 radians. (A1)

(c) $\begin{cases} x-7y=-1 \\ x-y+2z=0 \\ 2x+y+5z=2 \end{cases}$

$\rightarrow \begin{cases} x-7y=-1 \\ 6y+2z=1 \\ 15y+5z=4 \end{cases} (R_2-R_1 \ \& \ R_3-2R_1)$ Row operations (M1)(A1)

$\rightarrow \begin{cases} x-7y=-1 \\ 6y+2z=1 \\ 0=1.5 \end{cases} (R_3-2.5R_2)$

$0=1.5$, which leads to a contradiction. Contradiction (R1)

Thus, the three planes do not intersect. (AG)