Chapter 20 Solution

Quick Practice

Part I Solution

(a) (1) $a = -0.7484151116$, $b = 11.46800981$, $c = -57.93589471$,
$d = 450.8750458$
$a = -0.748$, $b = 11.5$, $c = -57.9$, $d = 451$

(2) The estimated number of books sold
$= -0.7484151116(6)^3 + 11.46800981(6)^2 - 57.93589471(6) + 450.8750458$
$= 354.4503666$
$= 354$

(3) This estimation is valid as this is an interpolation.

(b) (1) 346.2

(2) H_0: $\mu = 370$

(3) H_1: $\mu < 370$

(4) $p\text{-value} = 0.0312739104$
$p\text{-value} = 0.0313$

(5) The null hypothesis is rejected.
As $p\text{-value} < 0.05$.

Part II Solution

(c) (1) $y(n) = 403.9123539 \cdot 0.9791106773^n$

$y(n) = 403.91 \cdot 0.97911^n$

(2) $R^2 = 0.9709573534$

$R^2 = 0.971$

(3) 97.1% of the variability of the data is explained by the regression model.

(d) (1) 6

(2) 1.54

(3) The null hypothesis is not rejected.

As $\chi^2_{calc} < 14.449$.

Part III Solution

(e) (1) Let $\mu_d, d = y - x$ be the mean of the differences of the numbers of books sold by Olga and Nina.

$H_0: \mu_d = 0$

(2) $H_1: \mu_d > 0$

(3) $p\text{-value} = 0.0583641326$

$p\text{-value} = 0.0584$

(4) The null hypothesis is rejected.

As $p\text{-value} < 0.1$.

(f) (1) The numbers of books sold by the two authors follow a bivariate normal distribution.

(2) $H_0: \rho = 0$

(3) $H_1: \rho \neq 0$

(4) $p\text{-value} = 0.0111187257$

$p\text{-value} = 0.0111$

(5) The null hypothesis is rejected.

As $p\text{-value} < 0.05$.

(g) (1) 90% confidence interval:

(–0.592, 20.0)

(2) The above result is not consistent with the conclusion of the hypothesis test in (e) as 0 is included in the confidence interval.

Exercise 81

1. (a) $\frac{dv}{dx} = 4v + 12y$

$\therefore \frac{d}{dx}\left(\frac{dy}{dx}\right) = 4\frac{dy}{dx} + 12y$ M1

$\frac{d^2y}{dx^2} = 4\frac{dy}{dx} + 12y$ A1

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y = 0$ AG

[2]

(b) (1) $\mathbf{M} = \begin{pmatrix} 4 & 12 \\ 1 & 0 \end{pmatrix}$ A1

(2) The characteristic polynomial of $\mathbf{M}$

$= \det(\mathbf{M} - \lambda\mathbf{I})$

$= \begin{vmatrix} 4-\lambda & 12 \\ 1 & 0-\lambda \end{vmatrix}$ M1

$= (4-\lambda)(-\lambda) - (12)(1)$

$= \lambda^2 - 4\lambda - 12$ A1

(3) $\lambda_1 = -2$, $\lambda_2 = 6$ A2

(4) $\mathbf{v}_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ A2

(5) $y = Ae^{-2x} + Be^{6x}$ A2

(6) $\frac{dy}{dx} = A(e^{-2x})(-2) + B(e^{6x})(6)$ A1

$\frac{dy}{dx} = -2Ae^{-2x} + 6Be^{6x}$

$\frac{d^2y}{dx^2} = -2Ae^{-2x}(-2) + 6Be^{6x}(6)$ A1

$\frac{d^2y}{dx^2} = 4Ae^{-2x} + 36Be^{6x}$

$\therefore \frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y$

$= (4Ae^{-2x} + 36Be^{6x}) - 4(-2Ae^{-2x} + 6Be^{6x})$ M1

$-12(Ae^{-2x} + Be^{6x})$

$= 4Ae^{-2x} + 36Be^{6x} + 8Ae^{-2x} - 24Be^{6x}$

$-12Ae^{-2x} - 12Be^{6x}$

$= 0$

Thus, the general solution for y satisfies the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y = 0$. AG

[12]

(c) (1) The eigenvalues of **M** are the solutions of $\lambda^2 - 4\lambda - 12 = 0$, where -4 and -12 are the coefficients of $\frac{dy}{dx}$ and y in

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y = 0$. R2

(2) $\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ A2

[4]

(d) (1) $\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 27y = 0$

$\therefore \lambda^2 - 12\lambda + 27 = 0$ A1

$(\lambda - 3)(\lambda - 9) = 0$

$\lambda = 3$ or $\lambda = 9$ A1

$\therefore y = Ae^{3x} + Be^{9x}$ A1

(2) $3 = Ae^{3(0)} + Be^{9(0)}$

$3 = A + B$ A1

$\frac{dy}{dx} = A(e^{3x})(3) + B(e^{9x})(9)$ M1

$\frac{dy}{dx} = 3Ae^{3x} + 9Be^{9x}$

$15 = 3Ae^{3(0)} + 9Be^{9(0)}$

$15 = 3A + 9B$ A1

By solving this system, $A = 2$ and $B = 1$. A1

$\therefore y = 2e^{3x} + e^{9x}$ A1

[8]

2.	(a)	(1)	-2	A1	
		(2)	$\frac{1}{2}$	A1	
		(3)	The equation of the perpendicular bisector of QR :		
			$y-7=\frac{1}{2}(x-5)$	M1	
			$y-7=\frac{1}{2}x-\frac{5}{2}$		
			$y=\frac{1}{2}x+\frac{9}{2}$	A1	
		(4)	$y=\frac{1}{2}(10)+\frac{9}{2}$	M1	
			$y=\frac{19}{2}$		
			Thus, the coordinates of C are $\left(10, \frac{19}{2}\right)$.	A1	
		(5)	The required length of the highway		
			$=\sqrt{(10-5)^2+\left(\frac{19}{2}-7\right)^2}$	M1	
			$=5.590169944$	A1	
			$=559$ m	A1	
					[9]
	(b)	(1)	333 m	A1	
		(2)	The total length of the highway		
			$=559+500+333+2(527)+4(1000)$	M1A1	
			$=6446$ m		
			$=6450$ m	A1	
					[4]
	(c)	(1)	6	A1	
		(2)	4	A1	
					[2]
	(d)	Eulerian circuit does not exist.		A1	
		As not all vertices are of even degree.		A1	
					[2]

(e)	(1)	CJ	A1
	(2)	For any four edges correct	A1
		For all edges correct	A1
		1. Choose CJ of time 10	
		2. Choose HI of time 20	
		3. Choose DE of time 25	
		4. Choose FG of time 25	
		5. Choose AB of time 30	
		6. Choose BH of time 30	
		7. Choose BC of time 35	
		8. Choose AD of time 40	
		9. Choose GH of time 50	
		Thus, the minimum spanning tree is a tree containing the highways CJ, HI, DE, FG, AB, BH, BC, AD and GH.	A1
	(3)	265 s	A1

[5]

(f)	For any five edges correct	A1
	For any ten edges correct	A1
	1. Choose EF of time 65	
	2. Choose FG of time 25	
	3. Choose GH of time 50	
	4. Choose HB of time 30	
	5. Choose BC of time 35	
	6. Choose CB of time 35	
	7. Choose BA of time 30	
	8. Choose AD of time 40	
	9. Choose DA of time 40	
	10. Choose AG of time 60	
	11. Choose GH of time 50	
	12. Choose HI of time 20	
	13. Choose IJ of time 55	
	14. Choose JC of time 10	
	15. Choose CD of time 45	
	16. Choose DE of time 25	
	Thus, a possible route contains EF, FG, GH, HB, BC, CB, BA, AD, DA, AG, GH, HI, IJ, JC, CD and DE.	A2

[4]

(g)	AD, BC, GH	A3

[3]