AI SL Practice Set 1 Paper 1 Solution

1.   (a)   The area of the rectangle

$$= 462000000 \text{ cm}^2$$

$$= 4.62 \times 10^8 \text{ cm}^2 \qquad\qquad \text{A2} \quad \text{N2}$$

[2]

(b)   The percentage error

$$= \left| \frac{450000000 - 462000000}{462000000} \right| \times 100\% \qquad \text{(A1) for substitution}$$

$$= 2.597402597\%$$

$$= 2.60\% \qquad\qquad \text{A1} \quad \text{N2}$$

[2]

2.   (a) $u_{10} = 181$

$$\therefore 100 + (10 - 1)d = 181 \qquad\qquad \text{(A1) for correct equation}$$

$$9d = 81$$

$$d = 9 \qquad\qquad \text{A1} \quad \text{N2}$$

[2]

(b)   208   A1   N1

[1]

(c)   The total number of seats

$$= \frac{15}{2} \left[2(100) + (15 - 1)(9) \right] \qquad \text{(A1) for substitution}$$

$$= 2445 \qquad\qquad \text{A1} \quad \text{N2}$$

[2]

3.   (a)   The mean ball speed

$$= \frac{80 + 76 + 100 + 66 + 40 + 116 + 90 + 76}{8} \qquad \text{(A1) for correct formula}$$

$$= 80.5 \text{ kmh}^{-1} \qquad\qquad \text{A1} \quad \text{N2}$$

[2]

(b)   (i) 78 kmh^{-1} A1   N1

(ii) 21.3 kmh^{-1} A1   N1

(iii) 76 kmh^{-1} A1   N1

[3]

4. (a) $y > 250$ (M1) for setting inequality

$20x > 250$

$x > 12.5$

Thus, the minimum number of tickets is 13.   A1   N2

[2]

(b) $y = 90 + 5x$ A1   N1

[1]

(c) $20x = 90 + 5x$ (M1) for setting equation

$15x = 90$

$x = 6$ (A1) for correct value

The amount of money

$= 20(6)$

$= 120$ USD   A1   N3

[3]

5. (a) (i) $x = 5$ A2   N2

(ii) $y = 4$ A2   N2

[4]

(b) $f(x) = 0$

$\dfrac{2 - 4x}{5 - x} = 0$ (M1) for setting equation

$2 - 4x = 0$

$2 = 4x$

$x = \dfrac{1}{2}$ A1   N2

[2]

6. (a) H_0: The gender and the teaching subjects are independent.   A1   N1

[1]

(b) The expected number

$$= \frac{(35+10+65+45)(10+35)}{300}$$ A1

$$= \frac{(155)(45)}{300}$$

$$= 23.25$$ AG   N0

[1]

(c) The p-value

$= 0.00002306699185$ (A1) for correct value

$= 0.0000231$ A1   N2

[2]

(d) The null hypothesis is rejected.   A1

As the p-value is less than 5%.   R1   N2

[2]

7. (a) (i) $r = \dfrac{3}{4}$ A1   N1

(ii) $u_4 = 10368$ A1   N1

[2]

(b) $u_7 = 24576\left(\dfrac{3}{4}\right)^{7-1}$ (M1) for substitution

$u_7 = 4374$

$u_8 = 24576\left(\dfrac{3}{4}\right)^{8-1}$

$u_8 = 3280.5$

Thus, the smallest term in the sequence that is an integer is $u_7 = 4374$.   A1   N2

[2]

(c) S_{27}

$$= \frac{24576\left(\left(\dfrac{3}{4}\right)^{27} - 1\right)}{\dfrac{3}{4} - 1}$$ (A1) for substitution

$= 98262.38736$

$= 98300$ A1   N2

[2]

8. (a) The expected number
$$= (13)(0.25)$$ (A1) for substitution
$$= 3.25$$ A1   N2

[2]

(b) The variance
$$= (13)(0.25)(1 - 0.25)$$ (A1) for substitution
$$= 2.4375$$ A1   N2

[2]

(c) The required probability
$$= \binom{13}{8}(0.25)^8(1 - 0.25)^{13-8}$$ (A1) for substitution
$$= 0.0046602041$$
$$= 0.00466$$ A1   N2

[2]

9. (a) $\cos A\hat{B}C = \dfrac{AB^2 + BC^2 - AC^2}{2(AB)(BC)}$ (M1) for cosine rule

$\cos A\hat{B}C = \dfrac{28^2 + 41^2 - 32^2}{2(28)(41)}$ (A1) for substitution

$\cos A\hat{B}C = 0.6276132404$

$A\hat{B}C = 51.12574956°$

$A\hat{B}C = 51.1°$ A1   N3

[3]

(b) The area of the park
$$= \frac{1}{2}(AB)(BC)\sin A\hat{B}C$$ (M1) for area formula

$$= \frac{1}{2}(28)(41)\sin 51.12574956°$$ (A1) for substitution

$$= 446.873514 \, \text{m}^2$$

$$= 447 \, \text{m}^2$$ A1   N3

[3]

10. (a) (i) The gradient of L

$$= -1 \div \frac{5-1}{7-5}$$ (M1) for valid approach

$$= -1 \div 2$$

$$= -\frac{1}{2}$$ A1   N2

(ii) The equation of L:

$$y - 4 = -\frac{1}{2}(x-4)$$ (M1) for substitution

$$y = -\frac{1}{2}x + 6$$ A1   N2

[4]

(b) Kimberly's office is on the boundary separating the Voronoi cells of the restaurant B and the restaurant C, which is equidistant to them.   R1   N1

[1]

11. (a) By TVM Solver:

$$\boxed{\begin{array}{l} N = 120 \\ I\% = 3.3 \\ PV = 950000 \\ PMT = ? \\ FV = 0 \\ P/Y = 12 \\ C/Y = 12 \\ PMT : END \end{array}}$$ (M1)(A1) for correct values

$PMT = -9305.412721$

Thus, the amount of monthly payment is $9310.   A1   N3

[3]

(b) The total amount to be paid

$$= (9305.412721)(120)$$ (M1) for valid approach

$$= \$1116649.527$$

$$= \$1120000$$ A1   N2

[2]

(c) The amount of interest paid

$$= 1116649.527 - 950000$$ (M1) for valid approach

$$= \$166649.5265$$

$$= \$167000$$ A1   N2

[2]

12. (a) The amount of bacteria

$= 100 \times 2^8$ (A1) for correct approach

$= 25600$ A1 N2

[2]

(b) (i) $100 = a \times b^0$ (M1) for setting equation

$a = 100$ A1 N2

(ii) $25600 = 100 \times b^{24}$ (M1) for setting equation

$b^{24} = 256$

$b^{24} - 256 = 0$

By considering the graph of

$y = b^{24} - 256$, $b = 1.259921$.

$\therefore b = 1.26$ A1 N2

[4]

13. (a) $a = 1$, $b = \pi^{-0.1}$ A2 N2

[2]

(b) The estimate of $\int_0^{0.5} f(x)\mathrm{d}x$

$= \dfrac{1}{2}(0.1)\left[1 + \pi^{-0.5} + 2(\pi^{-0.1} + \pi^{-0.2} + \pi^{-0.3} + \pi^{-0.4})\right]$ (A2) for substitution

$= 0.3811259104$

$= 0.381$ A1 N3

[3]

(c) Overestimate A1 N1

[1]

14. (a) 150 A1 N1

[1]

(b) 15 A1 N1

[1]

(c) $y = a(x - (-5))(x - 15)$ (A1) for correct approach

$y = a(x + 5)(x - 15)$

$150 = a(0 + 5)(0 - 15)$

$150 = -75a$

$a = -2$ A1 N2

$\therefore y = -2(x + 5)(x - 15)$

$y = -2(x^2 - 10x - 75)$ (A1) for correct approach

$y = -2x^2 + 20x + 150$

$\therefore b = 20$ A1 N2

[4]