AI HL Practice Set 1 Paper 1 Solution

1. (a) The mean ball speed

$= \frac{80+76+100+66+40+116+90+76}{8}$ (A1) for correct formula

$= 80.5 \text{ kmh}^{-1}$ A1

[2]

(b) (i) 78 kmh^{-1} A1

(ii) 21.3 kmh^{-1} A1

(iii) 76 kmh^{-1} A1

[3]

2. (a) $u_{10} = 181$

$\therefore 100 + (10-1)d = 181$ (A1) for correct equation

$9d = 81$

$d = 9$ A1

[2]

(b) 208 A1

[1]

(c) The total number of seats

$= \frac{15}{2}\left[2(100) + (15-1)(9)\right]$ (A1) for substitution

$= 2445$ A1

[2]

3. (a) $\cos A\hat{B}C = \frac{AB^2 + BC^2 - AC^2}{2(AB)(BC)}$ (M1) for cosine rule

$\cos A\hat{B}C = \frac{28^2 + 41^2 - 32^2}{2(28)(41)}$ (A1) for substitution

$\cos A\hat{B}C = 0.6276132404$

$A\hat{B}C = 51.12574956^\circ$

$A\hat{B}C = 51.1^\circ$ A1

[3]

(b) The area of the park

$= \frac{1}{2}(AB)(BC)\sin A\hat{B}C$ (M1) for area formula

$= \frac{1}{2}(28)(41)\sin 51.12574956^\circ$ (A1) for substitution

$= 446.873514\ \text{m}^2$

$= 447\ \text{m}^2$ A1

[3]

4. (a) (i) The gradient of L

$= -1 \div \frac{5-1}{7-5}$ (M1) for valid approach

$= -1 \div 2$

$= -\frac{1}{2}$ A1

(ii) The equation of L:

$y - 4 = -\frac{1}{2}(x - 4)$ (M1) for substitution

$y = -\frac{1}{2}x + 6$ A1

[4]

(b) Kimberly's office is on the boundary separating the Voronoi cells of the restaurant B and the restaurant C, which is equidistant to them. R1

[1]

5. (a) The expected number

$= (13)(0.25)$ (A1) for substitution

$= 3.25$ A1

[2]

(b) The variance

$= (13)(0.25)(1-0.25)$ (A1) for substitution

$= 2.4375$ A1

[2]

(c) The required probability

$= \binom{13}{8}(0.25)^8(1-0.25)^{13-8}$ (A1) for substitution

$= 0.0046602041$

$= 0.00466$ A1

[2]

6. (a) (i) $y = 20 - 4x$ A1

(ii) $0 < x < 5$ A1

[2]

(b) $V = (4x)(2x)(20-4x)$ (M1) for valid approach

$V = 8x^2(20-4x)$

$V = 160x^2 - 32x^3$ A1

[2]

(c) By considering the graph of $V = 160x^2 - 32x^3$, the coordinates of the maximum point are $(3.3333342, 592.59259)$. (M1) for valid approach

Thus, the maximum volume is $593\,\text{cm}^3$. A1

[2]

7. (a) By TVM Solver:

$$\boxed{\begin{array}{l} N=120 \\ I\%=3.3 \\ PV=950000 \\ PMT=? \\ FV=0 \\ P/Y=12 \\ C/Y=12 \\ PMT:END \end{array}}$$

(M1)(A1) for correct values

$PMT=-9305.412721$

Thus, the amount of monthly payment is $\$9310$. A1

[3]

(b) The total amount to be paid

$=(9305.412721)(120)$ (M1) for valid approach

$=\$1116649.527$

$=\$1120000$ A1

[2]

(c) The amount of interest paid

$=1116649.527-950000$ (M1) for valid approach

$=\$166649.5265$

$=\$167000$ A1

[2]

8. (a) 150 A1

[1]

(b) 15 A1

[1]

(c) $y=a(x-(-5))(x-15)$ (A1) for correct approach

$y=a(x+5)(x-15)$

$150=a(0+5)(0-15)$

$150=-75a$

$a=-2$ A1

$\therefore y=-2(x+5)(x-15)$

$y=-2(x^2-10x-75)$ (A1) for correct approach

$y=-2x^2+20x+150$

$\therefore b=20$ A1

[4]

9.	(a)	(i)	420 g	A1	
		(ii)	243 g	A1	
					[2]
	(b)	(i)	1820 g	A1	
		(ii)	40.2 g	A1	
					[2]
	(c)	$Y \sim \mathrm{N}(1820, 1615)$ $\mathrm{P}(Y \geq 1770)$ $= 0.8932835503$ $= 0.893$		 (A1) for correct value A1	
					[2]
10.	(a)	$W = k\sqrt[3]{A}$, where $k \neq 0$ $96 = k\sqrt[3]{512}$ $k = 12$ $\therefore W = 12\sqrt[3]{A}$		(M1) for valid approach A1	
					[2]
	(b)	$125\ \mathrm{cm}^2$		A1	
					[1]
	(c)	Vertical stretch of scale factor 2 followed by translate upward by 7 units.		A1 A1	
					[2]

11. (a) $X \sim \text{Po}(\lambda)$

$\text{P}(X = 25) = 0.0555460$

$\text{P}(X = 25) - 0.0555460 = 0$ (A1) for correct approach

By considering the graph of

$y = \text{P}(X = 25) - 0.0555460$, $\lambda = 21.000003$.

$\therefore \lambda = 21$ A1

[2]

(b) (i) $\text{P}(X \geq 19)$

$= 1 - \text{P}(X \leq 18)$ (M1) for valid approach

$= 1 - 0.301680304$

$= 0.698319696$

$= 0.698$ A1

(ii) $Y \sim \text{Po}\left(\dfrac{21}{7}\right)$ (M1) for valid approach

$\text{P}(Y = 1)$

$= 0.1493612051$

$= 0.149$ A1

(iii) The required probability

$= 0.1493612051^4$ (M1) for valid approach

$= 0.0004976812006$

$= 0.000498$ A1

[6]

12.	(a)	By considering the graph of $y = 8e^t \sin 3t$, the maximum distance		(M1) for valid approach	
		$= 115.8163\text{ cm}$			
		$= 116\text{ cm}$		A1	
					[2]
	(b)	(i)	By considering the graph of $y = 8e^t \sin 3t$, the particle first goes back to O at 1.0471976 s.	(M1) for valid approach	
			Thus, the required time is 1.05 s.	A1	
		(ii)	$s'(t)$		
			$= (8e^t)(\sin 3t) + (8e^t)(3\cos 3t)$	(M1) for product rule	
			$= 8e^t(\sin 3t + 3\cos 3t)$	A1	
		(iii)	$s''(1.0471976)$		
			$= -136.783\text{ cms}^{-2}$		
			$= -137\text{ cms}^{-2}$	A1	
					[5]
13.	(a)	(i)	$H_0: \mu_d = 0$	A1	
		(ii)	$H_1: \mu_d < 0$	A1	
					[2]
	(b)	The p-value			
		$= 0.1427954705$		(A1) for correct value	
		$= 0.143$		A1	
					[2]
	(c)	The null hypothesis is not rejected.		A1	
		As $p\text{-value} > 0.05$.		R1	
					[2]

14. (a) $h(x) = g(f(x))$ (M1) for composite function

$h(x) = 2\sin\left(\frac{f(x)}{3}\right) - 6$ (A1) for substitution

$h(x) = 2\sin\left(\frac{9x+1}{3}\right) - 6$

$h(x) = 2\sin\left(3x + \frac{1}{3}\right) - 6$ A1

[3]

(b) The period of h

$= 2\pi \div 3$ (M1) for valid approach

$= \frac{2\pi}{3}$ A1

[2]

(c) $\{y : -8 \le y \le -4\}$ A2

[2]

15. (a) (i) 1 A1

(ii) $\frac{5}{16}$ A1

[2]

(b) $f(x) = a\left(x - \left(\frac{1}{2} + \frac{1}{4}\mathrm{i}\right)\right)\left(x - \left(\frac{1}{2} - \frac{1}{4}\mathrm{i}\right)\right)$ (M1) for valid approach

$f(x) = a\left(x^2 - \left(\left(\frac{1}{2} + \frac{1}{4}\mathrm{i}\right) + \left(\frac{1}{2} - \frac{1}{4}\mathrm{i}\right)\right)x + \left(\frac{1}{2} + \frac{1}{4}\mathrm{i}\right)\left(\frac{1}{2} - \frac{1}{4}\mathrm{i}\right)\right)$ (A1) for correct approach

$f(x) = a\left(x^2 - x + \frac{5}{16}\right)$ A1

[3]

(c) $\frac{5}{2} = a\left(1^2 - 1 + \frac{5}{16}\right)$ (M1) for setting equation

$\frac{5}{2} = \frac{5}{16}a$

$a = 8$ A1

[2]

16. (a) The required value

$=V(11)$

$=\frac{1000000}{1+29e^{-2.175}}(11+15)$ (M1) for substitution

$=\$6054063.077$

$=\$6050000$ A1

[2]

(b) $V(t)=10000000$

$\frac{30000000}{1+29e^{-0.145t}}=10000000$ (M1) for setting equation

$\frac{30000000}{1+29e^{-0.145t}}-10000000=0$

By considering the graph of

$y=\frac{30000000}{1+29e^{-0.145t}}-10000000$, $t=18.442404$.

$\therefore t=18.4$ A1

[2]

(c) The value of the pendulum clock will approach $30000000 after a long period of time. R1

[1]

17. (a) (i) $y=e^{0.25x}-1.25$

$y+1.25=e^{0.25x}$ M1

$\ln(y+1.25)=0.25x$ A1

$x=4\ln(y+1.25)$ AG

(ii) The area of R

$=\int_0^8 \left|4\ln(y+1.25)\right| \mathrm{d}y$ M1A1

$=49.19535365$

$=49.2$ A1

[5]

(b) The volume of the solid model

$=\int_0^8 \pi(4\ln(y+1.25))^2 \mathrm{d}y$ (A1) for correct approach

$=1061.499867$

$=1060$ A1

[2]

18. (a) A confidence interval with a smaller confidence level has a narrower interval about the mean. R1

[1]

(b) $(31.1, 44.9)$ A1

[1]

(c) $13.8 = 2(2.575829303)\left(\frac{\sigma}{\sqrt{11}}\right)$ M1A1

$\sigma = 8.884405122$ (A1) for correct value

$\therefore \sigma^2 = 78.93265438$

$\sigma^2 = 78.9$ A1

[4]