AI HL Practice Set 3 Paper 1 Solution

1. (a) $260-100=(31-11)d$ (M1) for valid approach

$160=20d$

$d=8$

Thus, the common difference is 8. A1

[2]

(b) $u_{11}=100$

$\therefore u_1+(11-1)(8)=100$ (A1) for correct equation

$u_1=20$ A1

[2]

(c) S_{51}

$=\frac{51}{2}\left[2(20)+(51-1)(8)\right]$ (A1) for substitution

$=11220$ A1

[2]

2. (a) 4 A1

[1]

(b) The inter-quartile range

$=6-2.5$ (M1) for valid approach

$=3.5$ A1

[2]

(c) The required probability

$=\frac{8}{12}$ (M1) for valid approach

$=\frac{2}{3}$ A1

[2]

3. (a) $\cos A\hat{C}B = \dfrac{AC^2 + BC^2 - AB^2}{2(AC)(BC)}$ (M1) for cosine rule

$\cos A\hat{C}B = \dfrac{54^2 + 54^2 - 35^2}{2(54)(54)}$ (A1) for substitution

$\cos A\hat{C}B = 0.789951989$

$A\hat{C}B = 37.81897498^\circ$

$A\hat{C}B = 37.8^\circ$ A1

[3]

(b) The required area

$= \dfrac{1}{2}(AC)(BC)\sin A\hat{C}B$ (M1) for area formula

$= \dfrac{1}{2}(54)(54)\sin 37.81897498^\circ$ (A1) for substitution

$= 893.999965 \text{ cm}^2$

$= 894 \text{ cm}^2$ A1

[3]

4. (a) $P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$ M1

$\therefore 5k^2 + (k^2 + 6k) + (k^2 + k) + k^2 = 1$ A1

$8k^2 + 7k - 1 = 0$

$(k + 1)(8k - 1) = 0$ A1

$k = -1$ (*Rejected*) or $k = \dfrac{1}{8}$ AG

[3]

(b) $P(X = 2 \mid X \le 2)$

$= \dfrac{P(X = 2 \cap X \le 2)}{P(X \le 2)}$

$= \dfrac{P(X = 2)}{P(X \le 2)}$ (M1) for valid approach

$= \dfrac{\left(\frac{1}{8}\right)^2 + 6\left(\frac{1}{8}\right)}{5\left(\frac{1}{8}\right)^2 + \left(\left(\frac{1}{8}\right)^2 + 6\left(\frac{1}{8}\right)\right)}$ (A1) for substitution

$= \dfrac{49}{54}$ A1

[3]

5. (a) $y = 5$ A1

[1]

(b) (i) $\left(5, \frac{7}{2}\right)$ A1

(ii) $k(5) + 2\left(\frac{7}{2}\right) - 47 = 0$ (M1) for substitution

$5k = 40$

$k = 8$ A1

(iii) $8x + 2(5) - 47 = 0$ (M1) for substitution

$8x = 37$

$x = \frac{37}{8}$

Thus, the required coordinates are

$\left(\frac{37}{8}, 5\right)$. A1

[5]

6. (a) $y = \frac{8}{7}$ A2

[2]

(b) $\left\{y : y \neq \frac{8}{7}, y \in \mathbb{R}\right\}$ A1

[1]

(c) $f(x) > g(x)$

$\frac{1-8x}{2-7x} > \frac{1}{2}x^2$

$\frac{1-8x}{2-7x} - \frac{1}{2}x^2 > 0$ M1

By considering the graph of $y = \frac{1-8x}{2-7x} - \frac{1}{2}x^2$,

$-1.439727 < x < 0.1239131$ or $\frac{2}{7} < x < 1.6015283$.

$\therefore -1.44 < x < 0.124$ or $\frac{2}{7} < x < 1.60$ A2

[3]

7. (a) Let $r\%$ be the nominal annual interest rate compounded yearly.

$$(1+r\%)^6 = \left(1+\frac{9}{(100)(12)}\right)^{(12)(6)}$$ (A1) for substitution

$1+r\% = 1.0075^{12}$

$r = 9.380689767$ (A1) for correct value

The real interest rate per year

$= 9.380689767\% - i\%$

$= (9.38069 - i)\%$ A1

[3]

(b) $$89000\left(1+\frac{9.38069-i}{100}\right)^6 = 118000$$ (M1) for setting equation

$$89000\left(1+\frac{9.38069-i}{100}\right)^6 - 118000 = 0$$ (A1) for correct approach

By considering the graph of

$$y = 89000\left(1+\frac{9.38069-i}{100}\right)^6 - 118000,$$

$i = 4.5676461$.

Thus, $i = 4.57$. A1

[3]

8. (a) The volume

$= \pi r^2 h$

$= \pi(4)^2(15)$ (A1) for substitution

$= 240\pi \text{ cm}^3$ A1

[2]

(b) The total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(4)^2 + 2\pi(4)(15)$ (A1) for substitution

$= 152\pi \text{ cm}^2$ A1

[2]

(c) 26 A1

[1]

9. (a) $f'(x)$

$= 0 + 9(2x) + 2(3x^2)$ (A1) for correct approach

$= 18x + 6x^2$ A1

[2]

(b) $f'(x) = 0$

$18x + 6x^2 = 0$

$6x(3 + x) = 0$ (A1) for factorization

$x = 0$ or $x = -3$ A1

[2]

(c) (i) $f''(x) = 18 + 12x$ A1

(ii) $f''(-3)$

$= 18 + 12(-3)$

$= -18 < 0$ R1

Therefore, f attains its local maximum at $x = -3$.

Thus, the x-coordinate of the local maximum of f is -3. A1

(iii) 57 A1

[4]

10. (a) H_0: The data follows a Poisson distribution with mean 3. A1

[1]

(b) 36.9 A1

[1]

(c) 5 A1

[1]

(c) 26.3 A2

[2]

(d) The null hypothesis is rejected. A1

As $\chi^2_{calc} > 11.070$. R1

[2]

11. (a) $\log\frac{1}{8}+\log\frac{1}{125}$

$=\log\left(\frac{1}{8}\cdot\frac{1}{125}\right)$ (A1) for correct formula

$=\log\frac{1}{1000}$

$=\log 10^{-3}$ (A1) for valid approach

$=-3$ A1

[3]

(b) $\ln e^{\frac{10}{3}}-\ln\sqrt[6]{e}$

$=\ln\frac{e^{\frac{10}{3}}}{e^{\frac{1}{6}}}$ (A1) for correct formula

$=\ln e^{\frac{10}{3}-\frac{1}{6}}$

$=\ln e^{\frac{19}{6}}$ (A1) for valid approach

$=\frac{19}{6}$ A1

[3]

12. (a) An unbiased estimate

$=\frac{700+698+\cdots+641}{12}$ (A1) for correct approach

$=663\text{ g}$ A1

[2]

(b) s_{n-1}

$=\sqrt{\frac{(700-663)^2+(698-663)^2+\cdots+(641-663)^2}{12-1}}$ (A1) for correct approach

$=31.53353194\text{ g}$

$=31.5\text{ g}$ A1

[2]

(c) 99% confidence interval:

$(634.73, 691.27)$ A2

[2]

13. (a) $g(x) = -f(x)$ (M1) for valid approach

$g(x) = -((x+1)^2 + 3)$

$g(x) = -(x+1)^2 - 3$ A1

[2]

(b) (i) $1 - p = -10$ (M1) for translation

$p = 11$ A1

(ii) $-3 + q = 0$ (M1) for translation

$q = 3$ A1

[4]

14. (a) $X \sim \text{Po}(1.75)$

$\text{P}(X \geq 3)$

$= 1 - \text{P}(X \leq 2)$ (M1) for valid approach

$= 1 - 0.7439696955$

$= 0.2560303045$

$= 0.256$ A1

[2]

(b) $Y \sim \text{Po}(12.25)$ (M1) for valid approach

$\text{P}(Y \leq 14)$

$= 0.7489477707$

$= 0.749$ A1

[2]

(c) The required probability

$= \text{P}(X \leq 2)^7$ (M1) for valid approach

$= 0.7439696955^7$

$= 0.1261498443$

$= 0.126$ A1

[2]

15. (a) By considering the graph of

$y = -x^3 + 17x^2 - 86x + 112$, $x = 2$, $x = 7$ or $x = 8$. (M1) for valid approach

Thus, the y-intercepts are 2, 7 and 8. A2

[3]

(b) The total area of the region

$= \int_2^8 \left| -y^3 + 17y^2 - 86y + 112 \right| \mathrm{d}y$ (A1) for correct approach

$= 73.83333519$

$= 73.8$ A1

[2]

16. (a) (i) 152.6 A1

(ii) 150.6 A1

(iii) 168.3 A1

[3]

(b) SS_{res}

$= (33\sqrt{24} - 160)^2 + (33\sqrt{26} - 160)^2$ (A1) for correct approach

$+ (33\sqrt{28} - 173)^2$

$= 73.75362941$

$= 73.8$ A1

[2]

(c) Model 2 A1

[1]

17. (a) $\mathbf{A}$

$= (\mathbf{A}^{-1})^{-1}$ (M1) for valid approach

$$= \begin{pmatrix} 2 & 1 & 1 \\ 2 & -3 & -5 \\ -1 & 1 & 3 \end{pmatrix}$$

$\therefore p = 2,\ q = 1$ A2

[3]

(b) $\begin{cases} 4x + 2y + 2z = 3 \\ x - 7y - 12z = 5 \\ x + 3y + 8z = 9 \end{cases}$ can be expressed as

$\begin{cases} 0.4x + 0.2y + 0.2z = 0.3 \\ 0.1x - 0.7y - 1.2z = 0.5 \\ 0.1x + 0.3y + 0.8z = 0.9 \end{cases}$. (M1) for valid approach

$$\mathbf{A}^{-1}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.5 \\ 0.9 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{A}\begin{pmatrix} 0.3 \\ 0.5 \\ 0.9 \end{pmatrix}$$ M1

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5.4 \\ 2.9 \end{pmatrix}$$

$\therefore x = 2,\ y = -5.4,\ z = 2.9$ A3

[5]

18. (a) $\sin 5x + \cos 4x = 0$

By considering the graph of $y = \sin 5x + \cos 4x$,

$x = 0.5235988$ or $x = 1.2217305$.

$\therefore x = 0.524$ or $x = 1.22$ A2

[2]

(b) $\sin 10x + \cos 8x$ is formed by a horizontal compression of $\sin 5x + \cos 4x$ of scale factor 2. R1

Therefore, there are still two distinct real roots when the range of x is halved at the same time. R1

Thus, the statement is incorrect. A1

[3]

(c) 6 A1

[1]