AI SL Practice Set 4 Paper 2 Solution

1. (a) The gradient of L_1

$= \frac{40-0}{0-30}$ (A1) for substitution

$= -\frac{4}{3}$ A1 N2

[2]

(b) The equation of L_1:

$y-40 = -\frac{4}{3}(x-0)$ (A1) for substitution

$3y-120 = -4x$

$4x+3y-120 = 0$ A1 N2

[2]

(c) The gradient of L_2

$= -1 \div -\frac{4}{3}$

$= \frac{3}{4}$ (A1) for correct value

The equation of L_2:

$y = \frac{3}{4}x$ A1 N2

[2]

(d) $4x+3\left(\frac{3}{4}x\right)-120 = 0$ (M1) for substitution

$6.25x = 120$

$x = 19.2$

$y = \frac{3}{4}(19.2)$ (M1) for substitution

$y = 14.4$

Thus, the coordinates of C are $(19.2, 14.4)$. A1 N3

[3]

(e) The area of the triangle OBC

$= \frac{(40-0)(19.2-0)}{2}$ (M1) for valid approach

$= 384$ A1 N2

[2]

(f) $BC = \sqrt{(0-19.2)^2 + (40-14.4)^2}$ (A1) for substitution

$BC = 32$ (A1) for correct value

$OC = \sqrt{(19.2-0)^2 + (14.4-0)^2}$

$OC = 24$ (A1) for correct value

The perimeter of the triangle OBC

$= 24 + 32 + 40$

$= 96$ A1 N4

[4]

(g) $\frac{3}{4}k$ A1 N1

[1]

(h) $\frac{(BC)(CD)}{2} = 624$ (A1) for correct equation

$32CD = 1248$

$CD = 39$ (A1) for correct value

$\therefore \sqrt{(k-19.2)^2 + \left(\frac{3}{4}k - 14.4\right)^2} = 39$ (A1) for correct equation

$\sqrt{(k-19.2)^2 + \left(\frac{3}{4}k - 14.4\right)^2} - 39 = 0$

By considering the graph of

$y = \sqrt{(k-19.2)^2 + \left(\frac{3}{4}k - 14.4\right)^2} - 39$, $k = -12$ or

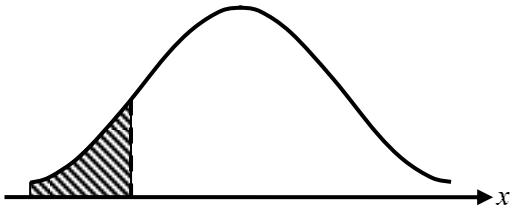
$k = 50.4$ (*Rejected*).

$\therefore k = -12$ A1 N4

[4]

2. (a) For vertical line clearly to the left of the mean A1
For shading to the left of the vertical line A1 N2

[2]



(b) (i) Let X be the volume of a randomly selected milk soda.
The required probability
$= \mathrm{P}(X < 490)$ (M1) for valid approach
$= 0.105649839$
$= 0.106$ A1 N2

(ii) The required probability
$= \mathrm{P}(X > 483 \mid X < 490)$ (M1) for valid approach
$= \dfrac{\mathrm{P}(X > 483 \cap X < 490)}{\mathrm{P}(X < 490)}$
$= \dfrac{\mathrm{P}(483 < X < 490)}{\mathrm{P}(X < 490)}$ (A1) for correct approach
$= 0.8410480651$
$= 0.841$ A1 N3

[5]

(c) The required probability
$= 2 \times \mathrm{P}(X < 490) \times (1 - \mathrm{P}(X < 490))$ (M1) for valid approach
$= 2 \times 0.105649839 \times (1 - 0.105649839)$ (A1) for substitution
$= 0.188975901$
$= 0.189$ A1 N3

[3]

(d) (i) 0.327 A2 N2

(ii) 0.0803 A2 N2

(iii) $-\$1.29$ A2 N2

[6]

3. (a) (i) $(6.67, 50.8)$ A2 N2

(ii) $2 < x < 6.67$ A2 N2

[4]

(b) (i) $f'(x) = -3x^2 + 13(2x) - 40(1) + 0$ (A1) for correct derivatives

$f'(x) = -3x^2 + 26x - 40$ A1 N2

(ii) 15 A1 N1

(iii) The equation of the tangent:

$y - f(5) = 15(x - 5)$ M1A1

$y - 36 = 15x - 75$ A1

$15x - y - 39 = 0$ AG N0

[6]

(c) (i) 9 A1 N1

(ii) $\int_2^9 f(x)\mathrm{d}x$ A1 N1

(iii) $\int_2^9 f(x)\mathrm{d}x = \frac{2401}{12}$ A2 N2

[4]

(d) The estimate of $\int_2^9 f(x)\mathrm{d}x$

$= \frac{1}{2}(1.75)\left[\begin{matrix} f(2) + f(9) \\ +2(f(3.75) + f(5.5) + f(7.25)) \end{matrix}\right]$ (A2) for substitution

$= \frac{1}{2}(1.75)\left[0 + 0 + 2\left(\begin{matrix} 16.078125 \\ +42.875 + 48.234375 \end{matrix}\right)\right]$ (A1) for correct approach

$= 187.578125$

$= 188$ A1 N4

[4]

(e) Underestimate A1 N1

[1]

4. (a) $\dfrac{\sin \hat{ACB}}{AB} = \dfrac{\sin \hat{ABC}}{AC}$ (M1) for sine rule

$\dfrac{\sin \hat{ACB}}{13.9} = \dfrac{\sin 60.8^\circ}{17.7}$ (A1) for substitution

$\hat{ACB} = 43.27612856^\circ$

$\hat{ACB} = 43.3^\circ$ A1 N3

[3]

(b) The area of the triangle ABC

$= \dfrac{1}{2}(AB)(AC)\sin \hat{BAC}$ (M1) for area formula

$= \dfrac{1}{2}(13.9)(17.7)\sin(180^\circ - 60.8^\circ - 43.27612856^\circ)$ (A1) for substitution

$= 119.3212815\ \text{cm}^2$

$= 119\ \text{cm}^2$ A1 N3

[3]

(c) $AB^2 = OA^2 + OB^2 - 2(OA)(OB)\cos \hat{AOB}$ (M1) for cosine rule

$13.9^2 = r^2 + r^2 - 2(r)(r)\cos(2(43.27612856^\circ))$ (A1) for substitution

$13.9^2 = 1.879723687r^2$ (A1) for correct approach

$r^2 = 102.7863836$

$r = 10.13836198$

$r = 10.1$ A1 N4

[4]

(d) The area of sector OAB

$= \pi(10.13836198)^2 \times \dfrac{2(43.27612856^\circ)}{360^\circ}$ (A1) for substitution

$= 77.63567911\ \text{cm}^2$

$= 77.6\ \text{cm}^2$ A1 N2

[2]

5. (a) 5.5 A1 N1

[1]

(b) $r_s = 0.8982196964$ (A1) for correct value

$r_s = 0.898$ A1 N2

[2]

(c) There is a strong agreement between the two experts. A1 N1

[1]

(d) (i) $a = 0.5610859729$

$a = 0.561$ A1 N1

$b = 11.53846154$

$b = 11.5$ A1 N1

(ii) The estimated percentage

$= 0.5610859729(50) + 11.53846154$ (A1) for substitution

$= 39.59276019\%$

$= 39.6\%$ A1 N2

[4]

(e) (i) $H_1 : \mu_x > \mu_y$ A1 N1

(ii) p-value $= 0.1727476756$ (A1) for correct value

p-value $= 0.173$ A1 N2

(iii) The null hypothesis is not rejected. A1

As p-value > 0.1. R1 N2

[5]