Chapter 17 Solution

Exercise 73

1.   (a)   The required probability

$$= \frac{2}{6} + \left(\frac{4}{6}\right)\left(\frac{4}{6}\right)\left(\frac{2}{6}\right) + \left(\left(\frac{4}{6}\right)\left(\frac{4}{6}\right)\right)^2 \left(\frac{2}{6}\right) \qquad \text{M1A1}$$

$$= \frac{1}{3} + \frac{4}{27} + \frac{16}{243}$$

$$= \frac{81}{243} + \frac{36}{243} + \frac{16}{243}$$

$$= \frac{133}{243} \qquad \text{A1}$$

[3]

      (b)   The required probability

$$= \frac{2}{6} + \left(\frac{4}{6}\right)\left(\frac{4}{6}\right)\left(\frac{2}{6}\right) + \left(\left(\frac{4}{6}\right)\left(\frac{4}{6}\right)\right)^2 \left(\frac{2}{6}\right) + \cdots \qquad \text{M1A1}$$

$$= \frac{1}{3}\left(1 + \frac{4}{9} + \left(\frac{4}{9}\right)^2 + \cdots\right)$$

$$= \frac{1}{3}\left(\frac{1}{1 - \frac{4}{9}}\right) \qquad \text{A1}$$

$$= \frac{1}{3}\left(\frac{9}{5}\right)$$

$$= \frac{3}{5} \qquad \text{A1}$$

[4]

2. (a) The required probability

$$= \left(\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\right)^2 \left(\frac{1}{5}\right)$$ M1A1

$$= \left(\frac{16}{25}\right)\left(\frac{16}{25}\right)\left(\frac{1}{5}\right)$$

$$= \frac{256}{3125}$$ A1

[3]

(b) The required probability

$$= \left(\frac{4}{5}\right)\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)$$

$$+ \left(\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\right)^2 \left(\frac{4}{5}\right)\left(\frac{1}{5}\right) + \cdots$$ M1A1

$$= \frac{4}{25}\left(1 + \frac{16}{25} + \left(\frac{16}{25}\right)^2 + \cdots\right)$$

$$= \frac{4}{25}\left(\frac{1}{1 - \frac{16}{25}}\right)$$ A1

$$= \frac{4}{25}\left(\frac{25}{9}\right)$$

$$= \frac{4}{9}$$ A1

[4]

3. (a) The required probability

$$= \frac{1}{5} + \left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)\left(\frac{1}{1}\right)$$ M1A1

$$= \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$$

$$= \frac{3}{5}$$ A1

[3]

(b) The required probability

$$= \frac{1}{5} + \left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{1}{5}\right) + \left(\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\right)^2\left(\frac{1}{5}\right) + \cdots$$ M1A1

$$= \frac{1}{5}\left(1 + \frac{16}{25} + \left(\frac{16}{25}\right)^2 + \cdots\right)$$

$$= \frac{1}{5}\left(\frac{1}{1 - \frac{16}{25}}\right)$$ A1

$$= \frac{1}{5}\left(\frac{25}{9}\right)$$

$$= \frac{5}{9}$$ A1

[4]

4. (a) The required probability

$$= \frac{1}{4} + \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right) + \left(\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\right)^2 \left(\frac{1}{4}\right) + \cdots \qquad \text{M1A1}$$

$$= \frac{1}{4}\left(1 + \frac{9}{16} + \left(\frac{9}{16}\right)^2 + \cdots\right)$$

$$= \frac{1}{4}\left(\frac{1}{1 - \frac{9}{16}}\right) \qquad \text{A1}$$

$$= \frac{1}{4}\left(\frac{16}{7}\right)$$

$$= \frac{4}{7} \qquad \text{A1}$$

[4]

(b) $$\left(\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\right)^{n-1}\left(\frac{3}{4}\right)\left(\frac{1}{4}\right) = \frac{243}{4096} \qquad \text{M1A1}$$

$$\left(\frac{9}{16}\right)^{n-1}\left(\frac{3}{16}\right) = \frac{243}{4096}$$

$$\left(\frac{9}{16}\right)^{n-1} = \frac{81}{256}$$

$$\left(\frac{9}{16}\right)^{n-1} = \left(\frac{9}{16}\right)^2 \qquad \text{(A1) for correct equation}$$

$$\therefore n - 1 = 2$$

$$n = 3 \qquad \text{A1}$$

[4]

1. (a) The required probability

$$= \left(\frac{50}{50+36}\right)(11\%) + \left(\frac{36}{50+36}\right)(17\%) \qquad \text{M1A1}$$

$$= \frac{581}{4300} \qquad \text{A1}$$

[3]

(b) The required probability

$$= \frac{\left(\dfrac{50}{50+36}\right)(11\%)}{\dfrac{581}{4300}} \qquad \text{M1A1}$$

$$= \frac{275}{581} \qquad \text{A1}$$

[3]

2. (a) Let p be the required probability.

$$p\left(\frac{11}{40}\right) + (1-p)\left(\frac{3}{40}\right) = 0.121 \qquad \text{M1A1}$$

$$\frac{11}{40}p + \frac{3}{40} - \frac{3}{40}p = 0.121$$

$$\frac{1}{5}p = 0.046$$

$$p = 0.23 \qquad \text{A1}$$

[3]

(b) The required probability

$$= \frac{(0.23)\left(1 - \dfrac{11}{40}\right)}{1 - 0.121} \qquad \text{M1A1}$$

$$= \frac{667}{3516} \qquad \text{A1}$$

[3]

3. (a) $(1.5\%)\left(\dfrac{100p}{100}\right)+(1-1.5\%)\left(\dfrac{p}{100}\right)=0.024353$ M1A1

$0.015p+0.00985p=0.024353$

$0.02485p=0.024353$

$p=0.98$ A1

[3]

(b) The required probability

$=\dfrac{(1-1.5\%)\left(\dfrac{0.98}{100}\right)}{0.024353}$ M1A1

$=\dfrac{197}{497}$ A1

[3]

4. The required probability

$=\dfrac{\left(\dfrac{2}{7}\right)(0.465)}{\left(\dfrac{5}{7}\right)(0.675)+\left(\dfrac{2}{7}\right)(0.465)}$ M1A2

$=\dfrac{62}{287}$ A1

[4]

1. (a) The required probability

$$=\left(\frac{35}{145}\right)\left(\frac{20}{35}\right)+\left(\frac{60}{145}\right)\left(\frac{18}{60}\right)+\left(\frac{145-35-60}{145}\right)\left(\frac{3}{5}\right)$$ M1A1

$$=\frac{68}{145}$$ A1

[3]

(b) The required probability

$$=\frac{\left(\dfrac{145-35-60}{145}\right)\left(\dfrac{3}{5}\right)}{\dfrac{68}{145}}$$ M1A1

$$=\frac{15}{34}$$ A1

[3]

2. (a) $(2p)(45\%)+(p)(15\%)+(1-2p-p)(8\%)=0.2015$ M1A1

$0.9p+0.15p+0.08-0.24p=0.2015$

$0.81p=0.1215$

$p=0.15$ A1

[3]

(b) The required probability

$$=\frac{(1-0.3-0.15)(1-8\%)}{1-0.2015}$$ M1A1

$$=\frac{1012}{1597}$$ A1

[3]

3. (a) $\left(\dfrac{50}{160}\right)(2q\%)+\left(\dfrac{160-50}{160}\right)(q\%)=0.105$ M1A1

$\left(\dfrac{50}{160}\right)\left(\dfrac{2q}{100}\right)+\left(\dfrac{110}{160}\right)\left(\dfrac{q}{100}\right)=0.105$

$\dfrac{21}{1600}q=0.105$

$q=8$ A1

[3]

(b) The required probability

$=\dfrac{\left(\dfrac{110}{160}\right)(1-12\%-8\%)}{\left(\dfrac{50}{160}\right)(1-8\%-16\%)+\left(\dfrac{110}{160}\right)(1-12\%-8\%)}$ M1A1

$=\dfrac{44}{63}$ A1

[3]

4. The required probability

$=\dfrac{\left(\dfrac{11}{7+11+2}\right)(1\%)+\left(\dfrac{2}{7+11+2}\right)(3\%)}{\left(\dfrac{7}{7+11+2}\right)(2\%)+\left(\dfrac{11}{7+11+2}\right)(1\%)+\left(\dfrac{2}{7+11+2}\right)(3\%)}$ M1A2

$=\dfrac{17}{31}$ A1

[4]

1.   (a)   The required probability

$$= \frac{\binom{7}{6} \times \binom{10}{2}}{\binom{17}{8}}$$ M1A1

$$= \frac{63}{4862}$$ A1

                                                          [3]

   (b)   The required probability

$$= \frac{\binom{7}{6} \times \binom{10}{2} + \binom{7}{7} \times \binom{10}{1}}{\binom{17}{8}}$$ M1A1

$$= \frac{5}{374}$$ A1

                                                          [3]

2.   (a)   The required probability

$$= \frac{\binom{15}{1} \times \binom{4}{4}}{\binom{19}{5}}$$ M1A1

$$= \frac{5}{3876}$$ A1

                                                          [3]

   (b)   The required probability

$$= \frac{\binom{15}{1} \times \binom{4}{4} + \binom{15}{3} \times \binom{4}{2} + \binom{15}{5} \times \binom{4}{0}}{\binom{19}{5}}$$ M1A1

$$= \frac{479}{969}$$ A1

                                                          [3]

3. (a) The required probability

$$= \frac{8! \times 6!}{13!}$$ (A2) for correct factorials

$$= \frac{2}{429}$$ A1

[3]

(b) The required probability

$$= \frac{^9P_5 \times 8!}{13!}$$ (A2) for correct factorials

$$= \frac{14}{143}$$ A1

[3]

4. The required probability

$$= \frac{\frac{5! \times 5!}{10}}{\frac{10!}{10}}$$ (A3) for correct factorials

$$= \frac{1}{252}$$ A1

[4]