AA HL Practice Set 1 Paper 2 Solution

Section A

1. (a)

$y = 3x + 7$

$\Rightarrow x = 3y + 7$ (A1) for correct approach

$3y = x - 7$

$y = \dfrac{x-7}{3}$

$\therefore f^{-1}(x) = \dfrac{x-7}{3}$ A1

[2]

(b)

$(f \circ g)(x)$

$= 3g(x) + 7$ (A1) for substitution

$= 3(2\sqrt{x}) + 7$

$= 6\sqrt{x} + 7$ A1

[2]

(c)

$(f \circ g)(529)$

$= 6\sqrt{529} + 7$ (M1) for substitution

$= 145$ A1

[2]

2. (a) The volume

$= \frac{1}{3}\pi r^2 h$ (M1) for valid approach

$= \frac{1}{3}\pi (18)^2 (18)$

$= 6107.256119$ (A1) for correct value

$= 6110$

$= 6.11 \times 10^3 \text{ cm}^3$ A1

[3]

(b) $V = 27\left(\frac{2}{3}\pi R^3\right)$ (M1) for setting equation

$16(6107.256119) = 18\pi R^3$ (A1) for substitution

$R^3 = 1728$

$R = 12$ A1

The ratio

$= 18 : 12$

$= 3 : 2$ A1

[4]

3. (a) $r = \frac{5.4}{4.5}$ (M1) for valid approach

$r = 1.2$ A1

[2]

(b) $S_{12} = \frac{4.5(1.2^{12} - 1)}{1.2 - 1}$ (A1) for substitution

$S_{12} = 178.1122601$

$S_{12} = 178$ A1

[2]

(c) $u_n < 678$

$4.5 \cdot 1.2^{n-1} < 678$

$4.5 \cdot 1.2^{n-1} - 678 < 0$ (M1) for valid approach

By considering the graph of $y = 4.5 \cdot 1.2^{n-1} - 678$,

$n < 28.50673$. A1

Thus, the greatest value of n is 28. A1

[3]

4. (a) $20P_1 - 17P_0 = 0$

$\therefore 20(P_0 e^{k(1)}) - 17P_0 = 0$ A1

$20e^k - 17 = 0$

$e^k = 0.85$ M1

$k = \ln 0.85$ AG

[2]

(b) $\frac{P_t}{P_0} \le 0.5$

$\therefore \frac{P_0 e^{(\ln 0.85)t}}{P_0} \le 0.5$ (A1) for correct inequality

$e^{(\ln 0.85)t} \le 0.5$ (A1) for correct approach

$(\ln 0.85)t \le \ln 0.5$

$(\ln 0.85)t - \ln 0.5 \le 0$ A1

By considering the graph of

$y = (\ln 0.85)t - \ln 0.5$, $t \ge 4.2650243$. (M1) for valid approach

Thus, the least number of whole years is 43. A1

[5]

5. (a) $\mathrm{AB}^2 = r^2 + r^2 - 2(r)(r)\cos 2\alpha$ A1

$\mathrm{AB}^2 = 2r^2 - 2r^2 \cos 2\alpha$

$\mathrm{AB} = \sqrt{2r^2 - 2r^2 \cos 2\alpha}$ A1

$\mathrm{AB} = \sqrt{2r^2(1 - \cos 2\alpha)}$

$\mathrm{AB} = r\sqrt{2(1 - \cos 2\alpha)}$ AG

[2]

(b) The arc length ACB

$= (r)(2\alpha)$ A1

$= 2r\alpha$

$\therefore P$

$= 2r\alpha + r\sqrt{2(1 - \cos 2\alpha)}$ M1

$= 2r\alpha + r\sqrt{2(1 - (1 - 2\sin^2 \alpha))}$ A1

$= 2r\alpha + r\sqrt{2(2\sin^2 \alpha)}$ A1

$= 2r\alpha + r\sqrt{4\sin^2 \alpha}$

$= 2r\alpha + 2r\sin\alpha$ A1

$= 2r(\alpha + \sin\alpha)$ AG

[5]

6. By using row operations, the system

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 5 & 7 & 1 & 2 \\ 32 & 24 & -17 & 5 \end{array}\right) \text{ is reduced to } \left(\begin{array}{ccc|c} 1 & 0 & -\frac{11}{8} & -\frac{1}{8} \\ 0 & 1 & \frac{9}{8} & \frac{3}{8} \\ 0 & 0 & 0 & 0 \end{array}\right).$$ (M1) for valid approach

$$y+\frac{9}{8}z=\frac{3}{8}$$

$$y=\frac{3}{8}-\frac{9}{8}z$$ A1

$$x-\frac{11}{8}z=-\frac{1}{8}$$

$$x=-\frac{1}{8}+\frac{11}{8}z$$ A1

Let $z=t$.

$$x=-\frac{1}{8}+\frac{11}{8}t,\ y=\frac{3}{8}-\frac{9}{8}t$$

Thus, the vector equation of the line of intersection is

$$\mathbf{r}=\begin{pmatrix} -\frac{1}{8} \\ \frac{3}{8} \\ 0 \end{pmatrix}+t\begin{pmatrix} \frac{11}{8} \\ -\frac{9}{8} \\ 1 \end{pmatrix}.$$ A2

[5]

7. (a) $\frac{1-x}{1+ax}$

$= (1-x)(1+ax)^{-1}$

$= (1-x)\left(1+(-1)(ax)+\frac{(-1)(-2)}{2!}(ax)^2+\cdots\right)$ M1A1

$= (1-x)(1-ax+a^2x^2+\cdots)$

$= 1-ax+a^2x^2-x+ax^2-a^2x^3+\cdots$

$= 1+(-a-1)x+(a^2+a)x^2+\cdots$ A1

[3]

(b) (i) $1+(a^2+a)=21$ (A1) for correct equation

$a^2+a-20=0$

$(a+5)(a-4)=0$

$a=-5$ (*Rejected*) or $a=4$ A1

(ii) -5 A1

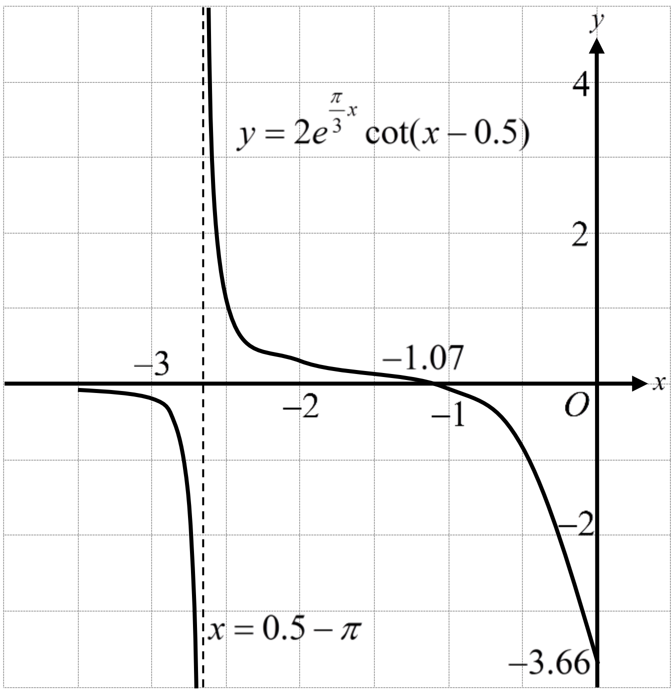
[3]

8. (a) For correct shape A1

For correct asymptote A1

For correct intercepts A1

[3]



(b) $0.0442 \le k \le 3.66$ A2

[2]

9. $y=4^{-x}$

$\log_4 y=-x$

$x=-\log_4 y$ (A1) for correct approach

$y=4^{-0}$

$y=1$ (A1) for correct value

$-\log_4 y=-\frac{1}{32}(y-24)^2$ (M1) for setting equation

$\frac{1}{32}(y-24)^2-\log_4 y=0$

By considering the graph of $x=\frac{1}{32}(y-24)^2-\log_4 y$,

$y=16$. (A1) for correct value

$0=-\frac{1}{32}(y-24)^2$

$0=(y-24)^2$

$y=24$ (A1) for correct value

The area of R

$=-\int_1^{16}(-\log_4 y)\mathrm{d}y-\int_{16}^{24}-\frac{1}{32}(y-24)^2\mathrm{d}y$ A1

$=26.51312053$

$=26.5$ A1

[7]

Section B

10. (a) The required probability

$= P(T \le 24)$ (M1) for valid approach

$= 0.9452007106$

$= 0.945$ A1

[2]

(b) $P(U \le 48) = 0.99494$

$P\left(Z \le \frac{48-\mu}{7}\right) = 0.99494$ (M1) for standardization

$\frac{48-\mu}{7} = 2.571701859$ A1

$48-\mu = 18.00191301$

$\mu = 29.99808699$

$\mu = 30.0$ A1

[3]

(c) The required probability

$= P(U \le 36)$ R1

$= 0.8043925789$ A1

Thus, for all school buses departing at $8:24\text{ am}$, 80.439% of them will arrive at school on time. AG

[2]

(d) The required probability

$= 1 - P(T \le 12)P(U \le 48)$

$-P(12 < T \le 24)P(U \le 36)$ M1A1

$= 1 - (0.2118553337)(0.99494)$

$-(0.7333453769)(0.80439)$ (A2) for correct values

$= 0.1993209666$

$= 0.199$ A1

[5]

(e) The expected number

$= (20)(0.1993209666)$ (A1) for correct formula

$= 3.986419331$

$= 3.99$ A1

[2]

11. (a) Let $\mathbf{n}_1$ and $\mathbf{n}_2$ be the normal vectors of the planes π_1 and π_2 respectively.

$\mathbf{n}_1 = \begin{pmatrix} 4 \\ 3 \\ k \end{pmatrix}$, $\mathbf{n}_2 = \begin{pmatrix} 4 \\ -3 \\ k \end{pmatrix}$ (A1) for correct values

$\mathbf{n}_1 \times \mathbf{n}_2 = \beta \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$

$\therefore \begin{pmatrix} (3)(k)-(k)(-3) \\ (k)(4)-(4)(k) \\ (4)(-3)-(3)(4) \end{pmatrix} = \beta \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$, where β is a constant. (A1) for substitution

$\begin{pmatrix} 6k \\ 0 \\ -24 \end{pmatrix} = \beta \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$

$\therefore \frac{6k}{-24} = \frac{-3}{1}$ A1

$k = 12$ A1

[4]

(b) (i) $a = 6$, $b = 8$, $c = 2$, $\alpha = -6$ A4

(ii) Let O be the origin.

The volume of the pyramid A′ABC

$= \frac{1}{3}\left(\frac{(\text{A}'\text{A})(\text{OB})}{2}\right)(\text{OC})$ (M1) for valid approach

$= \frac{1}{3}\left(\frac{(6-(-6))(8)}{2}\right)(2)$ A1

$= 32$ A1

[7]

(c) (i) $\overrightarrow{AC'} = -6\mathbf{i} - 2\mathbf{k}$

$\overrightarrow{AC'} \cdot (-\mathbf{i}) = \left|\overrightarrow{AC'}\right| \left|-\mathbf{i}\right| \cos C'\hat{A}A'$ (M1) for valid approach

$(-6\mathbf{i} - 2\mathbf{k}) \cdot (-\mathbf{i})$

$= (\sqrt{(-6)^2 + (-2)^2})(1) \cos C'\hat{A}A'$ (A1) for substitution

$(-6)(-1) + (-2)(0) = \sqrt{40} \cos C'\hat{A}A'$

$\cos C'\hat{A}A' = \frac{6}{\sqrt{40}}$

$C'\hat{A}A' = 18.43494882^\circ$

$C'\hat{A}A' = 18.4^\circ$ A1

(ii) $\because C'A' = C'A$

$\therefore C'\hat{A}'A = 18.43494882^\circ$ (A1) for correct approach

$A\hat{C}'A' + 18.43494882^\circ + 18.43494882^\circ = 180^\circ$

$A\hat{C}'A' = 143.1301024^\circ$

$A\hat{C}'A' = 143^\circ$ A1

[5]

(d) The vector equation of L:

$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + s \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix}$

$\begin{cases} x = 4s \\ y = 3s \\ z = 2 + 12s \end{cases}$ (A1) for correct approach

$\therefore 4(4s) - 3(3s) + 12(2 + 12s) = -24$ (A1) for substitution

$151s = -48$

$s = -\frac{48}{151}$

$\begin{cases} x = 4\left(-\frac{48}{151}\right) = -1.271523179 \\ y = 3\left(-\frac{48}{151}\right) = -0.9536423841 \\ z = 2 + 12\left(-\frac{48}{151}\right) = -1.814569536 \end{cases}$ M1

Thus, the coordinates of Q are

$(-1.2715, -0.9536, -1.8146)$. A1

[4]

12. (a) (i) $f'(x)=\left(\frac{1}{x^2+1}\right)(2x)$

$f'(x)=\frac{2x}{x^2+1}$ A1

$f''(x)=\frac{(x^2+1)(2)-(2x)(2x)}{(x^2+1)^2}$ (M1) for valid approach

$f''(x)=\frac{2x^2+2-4x^2}{(x^2+1)^2}$

$f''(x)=\frac{2-2x^2}{(x^2+1)^2}$ A1

$f^{(3)}(x)=\frac{(x^2+1)^2(-4x)-(2-2x^2)(2)(x^2+1)(2x)}{(x^2+1)^4}$ (M1) for valid approach

$f^{(3)}(x)=\frac{-4x^3-4x-8x+8x^3}{(x^2+1)^3}$

$f^{(3)}(x)=\frac{4x^3-12x}{(x^2+1)^3}$ A1

(ii) $f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)$

$+\frac{x^3}{3!}f^{(3)}(0)+\frac{x^4}{4!}f^{(4)}(0)+\cdots$

$f(x)=\ln(0^2+1)+x\left(\frac{2(0)}{0^2+1}\right)$

$+\frac{x^2}{2}\left(\frac{2-2(0)^2}{(0^2+1)^2}\right)+\frac{x^3}{6}\left(\frac{4(0)^3-12(0)}{(0^2+1)^3}\right)$ M2

$+\frac{x^4}{24}\left(-\frac{12(0^4-6(0)^2+1)}{(0^2+1)^4}\right)+\cdots$

$f(x)=0+x(0)+\frac{x^2}{2}(2)$

$+\frac{x^3}{6}(0)+\frac{x^4}{24}(-12)+\cdots$ A2

$f(x)=x^2-\frac{1}{2}x^4+\cdots$ A1

[10]

(b) $\sin x = x - \frac{x^3}{3!} + \cdots$

$\ln\left((x^2+1)^{\sin x}\right)$

$= \sin x \ln(x^2+1)$ (A1) for correct approach

$= \left(x - \frac{x^3}{6} + \cdots\right)\left(x^2 - \frac{1}{2}x^4 + \cdots\right)$ M1A1

$= x^3 - \frac{1}{2}x^5 - \frac{1}{6}x^5 + \cdots$ (M1) for valid approach

$= x^3 - \frac{2}{3}x^5 + \cdots$ A1

[5]

(c) The approximate value of the volume

$= \int_{0.7}^{1.3} \pi\left(y\sqrt{\ln\left((y^2+1)^{\sin y}\right)}\right)^2 \mathrm{d}y$ (M1) for valid approach

$= \int_{0.7}^{1.3} \pi y^2 \ln\left((y^2+1)^{\sin y}\right) \mathrm{d}y$

$\approx \int_{0.7}^{1.3} \pi y^2\left(y^3 - \frac{2}{3}y^5\right) \mathrm{d}y$ A1

$\approx \int_{0.7}^{1.3} \pi\left(y^5 - \frac{2}{3}y^7\right) \mathrm{d}y$ (M1) for valid approach

≈ 0.3452245902

≈ 0.345 A1

[4]