Exercise 5.1

(a) $h(2)$

$= g(f(2))$ — $g(f(2))$ (M1)

$= g(7)$

$= -1$ — (A1)

(b) $h'(x) = g'(f(x)) \cdot f'(x)$ — $g'(f(x)) \cdot f'(x)$ (M1)

$h'(2)$

$= g'(f(2)) \cdot f'(2)$

$= g'(7) \cdot f'(2)$ — $g'(7) \cdot f'(2)$ (A1)

$= (2)(1)$

$= 2$ — (A1)

(c) $\alpha(2)$

$= f(2)g(2)$ — $f(2)g(2)$ (M1)

$= (7)(1)$

$= 7$ — (A1)

(d) $\alpha'(x) = f'(x)g(x) + f(x)g'(x)$ — Product rule (M1)

$\alpha'(2)$

$= f'(2)g(2) + f(2)g'(2)$ — $f'(2)g(2) + f(2)g'(2)$ (A1)

$= (1)(1) + (7)(0)$

$= 1$ — (A1)

(e) $\beta(7)$

$= \dfrac{f(7)}{g(7)}$ — $\dfrac{f(7)}{g(7)}$ (M1)

$= \dfrac{2}{-1}$

$= -2$ — (A1)

(f) $\beta'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$ Quotient rule (M1)

$\beta'(7)$

$= \dfrac{f'(7)g(7) - f(7)g'(7)}{(g(7))^2}$ $\dfrac{f'(7)g(7) - f(7)g'(7)}{(g(7))^2}$ (A1)

$= \dfrac{(0)(-1) - (2)(2)}{(-1)^2}$

$= -4$ (A1)

Exercise 5.2

(a) $f'(x)$

$$= \frac{\left(\frac{\mathrm{d}}{\mathrm{d}x}(6x+24)\right)(x^2+3x)-(6x+24)\left(\frac{\mathrm{d}}{\mathrm{d}x}(x^2+3x)\right)}{(x^2+3x)^2}$$ Quotient rule (M1)

$$= \frac{(6)(x^2+3x)-(6x+24)(2x+3)}{(x^2+3x)^2}$$ 6 (A1) & $2x+3$ (A1)

$$= \frac{6x^2+18x-(12x^2+66x+72)}{(x^2+3x)^2}$$ $12x^2+66x+72$ (M1)

$$= \frac{-6x^2-48x-72}{(x^2+3x)^2}$$ $-6x^2-48x-72$ (A1)

$$= \frac{-6(x^2+8x+12)}{(x^2+3x)^2}$$

$$= -\frac{6(x+2)(x+6)}{(x^2+3x)^2}$$ (AG)

(b) $f'(x)=0$

$$\therefore -\frac{6(x+2)(x+6)}{(x^2+3x)^2}=0$$ $-\frac{6(x+2)(x+6)}{(x^2+3x)^2}=0$ (M1)

$(x+2)(x+6)=0$ $(x+2)(x+6)=0$ (A1)

$x+2=0$ or $x+6=0$

$x=-2$ or $x=-6$ (*Rejected*) $x=-2$ (A1)

$f(-2)$

$$= \frac{6(-2)+24}{(-2)^2+3(-2)}$$ $x=-2$ (M1)

$=-6$

Thus, the coordinates of A are $(-2,-6)$. (A1)

Exercise 5.3

(a) $(x+2+h)^2$

$=(x+2+h)(x+2+h)$

$=x^2+2x+hx+2x+4+2h+hx+2h+h^2$ Nine terms (M1)

$=x^2+(4+2h)x+(4+4h+h^2)$ (A1)

(b) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

$=\lim_{h\to 0}\dfrac{(x+2+h)^2-(x+2)^2}{h}$

$=\lim_{h\to 0}\dfrac{x^2+(4+2h)x+(4+4h+h^2)-(x^2+4x+4)}{h}$ $x^2+(4+2h)x+(4+4h+h^2)$ (A1)

$=\lim_{h\to 0}\dfrac{x^2+4x+2hx+4+4h+h^2-x^2-4x-4}{h}$

$=\lim_{h\to 0}\dfrac{2hx+4h+h^2}{h}$

$=\lim_{h\to 0}(2x+4+h)$ $2x+4+h$ (A1)

$=2x+4+0$

$=2x+4$ (A1)

(c) $(y+2)^2-3e^x=5+ye^x$

$\frac{d}{dx}((y+2)^2)-\frac{d}{dx}(3e^x)=\frac{d}{dx}(5)+\frac{d}{dx}(ye^x)$ Implicit differentiation (M1)

$(2y+4)\left(\frac{dy}{dx}\right)-3(e^x)=0+\left(\frac{dy}{dx}\right)(e^x)+(y)(e^x)$ $(2y+4)(y')$ (A1), $3(e^x)$ (A1) & $(y')(e^x)+(y)(e^x)$ (A1)

$(2y+4)\frac{dy}{dx}-3e^x=e^x\frac{dy}{dx}+ye^x$

$(2y+4)\frac{dy}{dx}-e^x\frac{dy}{dx}=ye^x+3e^x$ $(2y+4)\frac{dy}{dx}-e^x\frac{dy}{dx}$ (M1)

$\frac{dy}{dx}(2y+4-e^x)=ye^x+3e^x$

$\frac{dy}{dx}=\frac{ye^x+3e^x}{2y+4-e^x}$ $\frac{ye^x+3e^x}{2y+4-e^x}$ (A1)

The gradient of the tangent

$=\frac{dy}{dx}$ at $(0,1)$

$=\frac{1e^0+3e^0}{2(1)+4-e^0}$ $x=0$ & $y=1$ (M1)

$=\frac{1+3}{2+4-1}$

$=\frac{4}{5}$ (A1)

Exercise 5.4

(a)	$f(1)$	
	$=-2e^{-1}$	
	$=-\dfrac{2}{e}$	$-\dfrac{2}{e}$ (A1)
	$f'(x)$	
	$=-2(e^{-x})(-1)$	Chain rule (M1)
	$=2e^{-x}$	
	The slope of the tangent	
	$=f'(1)$	$f'(1)$ (M1)
	$=2e^{-1}$	
	$=\dfrac{2}{e}$	$\dfrac{2}{e}$ (A1)
	The equation of the tangent:	
	$y-\left(-\dfrac{2}{e}\right)=\dfrac{2}{e}(x-1)$	$y-y_1=m(x-x_1)$ (M1)
	$y+\dfrac{2}{e}=\dfrac{2}{e}x-\dfrac{2}{e}$	
	$ey+2=2x-2$	
	$2x-ey-4=0$	(A1)
(b)	The slope of the normal	
	$=-1\div\dfrac{2}{e}$	
	$=-\dfrac{e}{2}$	$-\dfrac{e}{2}$ (A1)
	The equation of the normal:	
	$y-\left(-\dfrac{2}{e}\right)=-\dfrac{e}{2}(x-1)$	$y-y_1=m(x-x_1)$ (M1)
	$y+\dfrac{2}{e}=-\dfrac{e}{2}x+\dfrac{e}{2}$	
	$2ey+4=-e^2x+e^2$	
	$e^2x+2ey+(4-e^2)=0$	(A1)

Exercise 5.5

(a) $Q = t^3 - 12t^2 + 36t$

$\frac{dQ}{dt}$

$= 3t^2 - 12(2t) + 36(1)$ — $3t^2$, $2t$ & 1 (A1)

$= 3t^2 - 24t + 36$

$\frac{dQ}{dt} = 0$ — $\frac{dQ}{dt} = 0$ (M1)

$3t^2 - 24t + 36 = 0$

$t^2 - 8t + 12 = 0$

$(t-2)(t-6) = 0$

$t - 2 = 0$ or $t - 6 = 0$

$t = 2$ or $t = 6$ — 2 & 6 (A1)

By the first derivative test, — Checking $\frac{dQ}{dt}$ (M1)

t	$t<2$	$t=2$	$2<t<6$	$t=6$	$t>6$
$\frac{dQ}{dt}$	$+$	0	$-$	0	$+$

Thus, Q attains its maximum at $t = 2$. — (A1)

(b) 32 — (A1)

(c) $Q = t^3 - 12t^2 + \alpha t$

$\frac{dQ}{dt} = \frac{d}{dt}(t^3) - \frac{d}{dt}(12t^2) + \frac{d}{dt}(\alpha t)$ — Implicit differentiation (M1)

$\frac{dQ}{dt} = 3t^2 - 12(2t) + \left(\frac{d\alpha}{dt}\right)(t) + (\alpha)(1)$ — $3t^2, 12(2t)$ (A1) & Product rule (A1)

$\frac{dQ}{dt} = 3t^2 - 24t + t\frac{d\alpha}{dt} + \alpha$

The required rate of change

$= 3(1)^2 - 24(1) + (1)(-0.25) + 18$ — $\frac{dQ}{dt}$ (A1)

$= -3.25$ — (A1)

Exercise 5.6

(a) By considering the graph of $y = 6\sin t - t^3 \cos t$, the coordinates of the maximum point are $(3.7435506, 39.843756)$. — GDC approach (M1)

Thus, the maximum distance is 39.8 cm. — (A1)

(b) The particle first changes direction at 3.7435506 s. — 1.5707983 s (A1)

By considering the graph of $y = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}}{\mathrm{d}t}(6\sin t - t^3 \cos t)\right)$, the graph passes through the point $(3.7435506, -68.94427)$. — GDC approach (M1)

Thus, the acceleration is -68.9 cms^{-2}. — (A1)

Exercise 5.7

$f(x)$

$= \int \cos 2x \sin^3 2x \mathrm{d}x$ — Indefinite integral (M1)

Let $u = \sin 2x$.
$\frac{\mathrm{d}u}{\mathrm{d}x} = 2\cos 2x \Rightarrow \frac{1}{2}\mathrm{d}u = \cos 2x\mathrm{d}x$

Substitution (A1)

$= \int u^3 \cdot \frac{1}{2}\mathrm{d}u$ — $\int u^3 \cdot \frac{1}{2}\mathrm{d}u$ (A1)

$= \frac{1}{2}\left(\frac{1}{4}u^4\right) + C$

$= \frac{1}{8}u^4 + C$

$\therefore f(x) = \frac{1}{8}\sin^4 2x + C$ — $\frac{1}{8}\sin^4 2x + C$ (A1)

$3 = \frac{1}{8}\left(\sin\left(2\left(\frac{\pi}{2}\right)\right)\right)^4 + C$ — $x = \frac{\pi}{2}$ & $y = 3$ (M1)

$3 = \frac{1}{8}(0) + C$

$C = 3$

$\therefore f(x) = \frac{1}{8}\sin^4 2x + 3$ — (A1)

Exercise 5.8

(a) The area of R

$= \int_0^1 f(x)\,\mathrm{d}x$ — Definite integral (M1)

$= \int_0^1 \sqrt{\pi x + 1}\,\mathrm{d}x$

Let $u = \pi x + 1$.

$\frac{\mathrm{d}u}{\mathrm{d}x} = \pi \Rightarrow \frac{1}{\pi}\mathrm{d}u = \mathrm{d}x$

$x = 1 \Rightarrow u = \pi(1) + 1 = \pi + 1$

$x = 0 \Rightarrow u = \pi(0) + 1 = 1$

Substitution (A1)

$= \int_1^{\pi+1} \sqrt{u} \cdot \frac{1}{\pi}\,\mathrm{d}u$ — $\int_1^{\pi+1} \sqrt{u} \cdot \frac{1}{\pi}\,\mathrm{d}u$ (A1)

$= \frac{1}{\pi}\int_1^{\pi+1} u^{\frac{1}{2}}\,\mathrm{d}u$

$= \frac{1}{\pi}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^{\pi+1}$ — $\frac{2}{3}u^{\frac{3}{2}}$ (A1)

$= \frac{1}{\pi}\left(\frac{2}{3}(\pi+1)^{\frac{3}{2}} - \frac{2}{3}(1)^{\frac{3}{2}}\right)$

$= \frac{2}{3\pi}\left((\pi+1)^{\frac{3}{2}} - 1\right)$ — (A1)

(b) The required volume

$= \pi\int_0^1 (f(x))^2\,\mathrm{d}x$ — Definite integral (M1)

$= \pi\int_0^1 (\sqrt{\pi x + 1})^2\,\mathrm{d}x$

$= \pi\int_0^1 (\pi x + 1)\,\mathrm{d}x$ — $\pi\int_0^1 (\pi x + 1)\,\mathrm{d}x$ (A1)

$= \pi\left[\pi\left(\frac{1}{2}x^2\right) + x\right]_0^1$ — $\pi\left(\frac{1}{2}x^2\right) + x$ (A1)

$= \pi\left[\frac{\pi}{2}x^2 + x\right]_0^1$

$= \pi\left(\left(\frac{\pi}{2}(1)^2 + 1\right) - \left(\frac{\pi}{2}(0)^2 + 0\right)\right)$

$= \pi\left(\frac{\pi}{2} + 1\right)$

$= \frac{\pi(\pi+2)}{2}$ — (A1)

Exercise 5.9

(a) The velocity

$= v(3)$ — $v(3)$ (M1)

$= 3^2 e^{-0.5(3)}$

$= 2.008171441$ ms^{-1}

$= 2.01$ ms^{-1} — (A1)

(b) $v(t) = 1$ — $v(t) = 1$ (M1)

$t^2 e^{-0.5t} = 1$

$t^2 e^{-0.5t} - 1 = 0$

By considering the graph of $y = t^2 e^{-0.5t} - 1$, the horizontal intercepts are 1.4296118 and 8.6131695. — GDC approach (M1)

$\therefore t = 1.43$ or $t = 8.61$ — (A1)

(c) The total distance travelled

$= \int_0^{10} |v(t)| \, \mathrm{d}t$ — $\int_{t_1}^{t_2} |v(t)| \, \mathrm{d}t$ (M1)

$= \int_0^{10} |t^2 e^{-0.5t}| \, \mathrm{d}t$ — $\int_0^{10} |t^2 e^{-0.5t}| \, \mathrm{d}t$ (M1)

$= 14.00556769$ m

$= 14.0$ m — (A1)

(d) $a(t)$

$= v'(t)$ — $v'(t)$ (M1)

$= (2t)(e^{-0.5t}) + (t^2)(e^{-0.5t})(-0.5)$ — Product rule (M1)

$= 2te^{-0.5t} - 0.5t^2 e^{-0.5t}$ — (A1)

(e) By considering the graphs of $y = t^2 e^{-0.5t}$ and $y = \frac{\mathrm{d}}{\mathrm{d}t}(t^2 e^{-0.5t})$, at least one graph is below the horizontal axis for $4 \le t \le 10$. — GDC approach (M1)

$\therefore 4 \le t \le 10$ — 4 (A1) & 10 (A1)

Exercise 5.10

$$\lim_{x \to 0} \frac{\sec x - 1}{\sin 2x + \cos 2x - 1 - 2x}$$

$$= \lim_{x \to 0} \frac{\sec x \tan x - 0}{(\cos 2x)(2) + (-\sin 2x)(2) - 0 - 2(1)} \quad \left(\because \frac{0}{0} \right)$$

L'Hôpital's rule (M1)

$$= \lim_{x \to 0} \frac{\sec x \tan x}{2\cos 2x - 2\sin 2x - 2}$$

$\sec x \tan x$ (A1) &
$2\cos 2x - 2\sin 2x - 2$ (A1)

$$= \lim_{x \to 0} \frac{(\sec x \tan x)(\tan x) + (\sec x)(\sec^2 x)}{2(-\sin 2x)(2) - 2(\cos 2x)(2) - 0} \quad \left(\because \frac{0}{0} \right)$$

L'Hôpital's rule (M1)

$$= \lim_{x \to 0} \frac{\sec x \tan^2 x + \sec^3 x}{-4\sin 2x - 4\cos 2x}$$

$\sec x \tan^2 x + \sec^3 x$ (A1) &
$-4\sin 2x - 4\cos 2x$ (A1)

$$= \frac{\sec 0 \tan^2 0 + \sec^3 0}{-4\sin(2(0)) - 4\cos(2(0))}$$

$x = 0$ (M1)

$$= \frac{1}{-4}$$

$$= -\frac{1}{4}$$

(A1)

Exercise 5.11

(a) $\sin x = x - \frac{x^3}{3!} + \cdots$

$\sin 2x = 2x - \frac{(2x)^3}{3!} + \cdots$ Substitute by $2x$ (M1)

$\sin 2x = 2x - \frac{4}{3}x^3 + \cdots$ (A1)

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

$\cos 2x = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \cdots$ Substitute by $2x$ (M1)

$\cos 2x = 1 - 2x^2 + \frac{2}{3}x^4 - \cdots$ (A1)

(b) $\sin 4x$

$= 2\sin 2x \cos 2x$

$= 2\left(2x - \frac{4}{3}x^3 + \cdots\right)\left(1 - 2x^2 + \frac{2}{3}x^4 - \cdots\right)$ Series for $\sin 2x$ & $\cos 2x$ (M1)

$= 2\left(2x - 4x^3 - \frac{4}{3}x^3 + \cdots\right)$

$= 2\left(2x - \frac{16}{3}x^3 + \cdots\right)$ $2x - \frac{16}{3}x^3 + \cdots$ (A1)

$= 4x - \frac{32}{3}x^3 + \cdots$ (A1)

(c) $\sin\left(4\left(\frac{\pi}{16}\right)\right)=4\left(\frac{\pi}{16}\right)-\frac{32}{3}\left(\frac{\pi}{16}\right)^3+\cdots$ $x=\frac{\pi}{16}$ (M1)

$\sin\left(4\left(\frac{\pi}{16}\right)\right)<4\left(\frac{\pi}{16}\right)$

$\sin\frac{\pi}{4}<\frac{\pi}{4}$ $\frac{\pi}{4}$ (A1)

$\sin\left(4\left(\frac{\pi}{16}\right)\right)>4\left(\frac{\pi}{16}\right)-\frac{32}{3}\left(\frac{\pi}{16}\right)^3$

$\sin\frac{\pi}{4}>\frac{\pi}{4}-\frac{\pi^3}{384}$ $\frac{\pi}{4}-\frac{\pi^3}{384}$ (A1)

$\therefore \frac{\pi}{4}-\frac{\pi^3}{384}<\sin\frac{\pi}{4}<\frac{\pi}{4}$ Inequality for $\sin\frac{\pi}{4}$ (M1)

$\frac{\pi}{4}-\frac{\pi^3}{384}<\frac{\sqrt{2}}{2}<\frac{\pi}{4}$

$\frac{\pi}{2}-\frac{\pi^3}{192}<\sqrt{2}<\frac{\pi}{2}$ Inequality for $\sqrt{2}$ (M1)

$\frac{96\pi}{192}-\frac{\pi^3}{192}<\sqrt{2}<\frac{\pi}{2}$

$\frac{\pi(96-\pi^2)}{192}<\sqrt{2}<\frac{\pi}{2}$ (AG)

(d) $\lim_{x\to 0}\frac{3\arctan x}{f(x)}$

$=\lim_{x\to 0}\frac{3\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots\right)}{4x-\frac{32}{3}x^3+\cdots}$ Series for $\arctan x$ & $f(x)$ (M1)

$=\lim_{x\to 0}\frac{3x-x^3+\cdots}{4x-\frac{32}{3}x^3+\cdots}$ $\frac{3x-x^3+\cdots}{4x-\frac{32}{3}x^3+\cdots}$ (A1)

$=\lim_{x\to 0}\frac{3-x^2+\cdots}{4-\frac{32}{3}x^2+\cdots}$ $\frac{3-x^2+\cdots}{4-\frac{32}{3}x^2+\cdots}$ (A1)

$=\frac{3-0+\cdots}{4-0+\cdots}$ $x=0$ (M1)

$=\frac{3}{4}$ (AG)

Exercise 5.12

(a) $\frac{dy}{dx}=\frac{\sqrt{y}}{x^2}$

$\frac{1}{\sqrt{y}}dy=\frac{1}{x^2}dx$

$\int y^{-\frac{1}{2}}dy=\int x^{-2}dx$ Separating variables (M1)

$\frac{1}{\frac{1}{2}}y^{\frac{1}{2}}=\frac{1}{-1}x^{-1}+C$ $\frac{1}{\frac{1}{2}}y^{\frac{1}{2}}$ & $\frac{1}{-1}x^{-1}$ (A1)

$2y^{\frac{1}{2}}=-\frac{1}{x}+C$

$y^{\frac{1}{2}}=-\frac{1}{2x}+C$ $y^{\frac{1}{2}}=-\frac{1}{2x}+C$ (A1)

$y=\left(-\frac{1}{2x}+C\right)^2$ (A1)

(b)

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = y + e^x$	
$\dfrac{\mathrm{d}y}{\mathrm{d}x} - y = e^x$	$\dfrac{\mathrm{d}y}{\mathrm{d}x} - y = e^x$ (M1)
Integrating factor	
$= e^{\int -1\mathrm{d}x}$	$e^{\int -1\mathrm{d}x}$ (M1)
$= e^{-x}$	e^{-x} (A1)
$\therefore e^{-x}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) - e^{-x}y = e^{-x}(e^x)$	
$e^{-x}\dfrac{\mathrm{d}y}{\mathrm{d}x} - e^{-x}y = 1$	
$\dfrac{\mathrm{d}}{\mathrm{d}x}(ye^{-x}) = 1$	$\dfrac{\mathrm{d}}{\mathrm{d}x}(ye^{-x}) = 1$ (A1)
$ye^{-x} = \int 1\mathrm{d}x$	
$\dfrac{y}{e^x} = x + C$	
$y = e^x(x + C)$	$e^x(x + C)$ (A1)
$e^2 = e^2(2 + C)$	$y = e^2$ & $x = 2$ (M1)
$C = -1$	
$\therefore y = e^x(x - 1)$	(A1)

(c) $\frac{dy}{dx}=\frac{y}{x}+\frac{x^2}{y^2}$

Let $y=vx$.
$\frac{dy}{dx}=\left(\frac{dv}{dx}\right)(x)+(v)(1)=v+x\frac{dv}{dx}$ — Substitution (A1)

$v+x\frac{dv}{dx}=\frac{vx}{x}+\frac{x^2}{(vx)^2}$ — $v+x\frac{dv}{dx}=\frac{vx}{x}+\frac{x^2}{(vx)^2}$ (M1)

$v+x\frac{dv}{dx}=v+\frac{1}{v^2}$

$x\frac{dv}{dx}=\frac{1}{v^2}$ — $x\frac{dv}{dx}=\frac{1}{v^2}$ (A1)

$v^2 dv=\frac{1}{x}dx$

$\int v^2 dv=\int\frac{1}{x}dx$ — Separating variables (M1)

$\frac{1}{3}v^3=\ln x+C$ — $\frac{1}{3}v^3$ & $\ln x$ (A1)

$v^3=3\ln x+C$

$v=(3\ln x+C)^{\frac{1}{3}}$ — $v=(3\ln x+C)^{\frac{1}{3}}$ (A1)

$\frac{y}{x}=(3\ln x+C)^{\frac{1}{3}}$

$y=x(3\ln x+C)^{\frac{1}{3}}$

$3=1(3\ln 1+C)^{\frac{1}{3}}$ — $y=3$ & $x=1$ (M1)

$3=C^{\frac{1}{3}}$

$C=27$

$\therefore\ y=x(3\ln x+27)^{\frac{1}{3}}$ — (A1)

(d) $\begin{cases} x_{n+1} = x_n + 0.1 \\ y_{n+1} = y_n + 0.1 \left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_{(x_n,\, y_n)} \end{cases}$

$\begin{cases} x_{n+1} = x_n + 0.1 \\ y_{n+1} = y_n + 0.1\sqrt{\dfrac{x_n}{y_n}} \end{cases}$ $\begin{cases} x_{n+1} = x_n + 0.1 \\ y_{n+1} = y_n + 0.1\sqrt{\dfrac{x_n}{y_n}} \end{cases}$ (M1)

$x_0 = 1$, $y_0 = 3$

By solving the system $\begin{cases} x_{n+1} = x_n + 0.1 \\ y_{n+1} = y_n + 0.1\sqrt{\dfrac{x_n}{y_n}} \end{cases}$ with

initial conditions $x_0 = 1$ and $y_0 = 3$, $x_8 = 1.8$ and

$y_8 = 3.5157923910534$. GDC approach (M1)

$\therefore$ $y = 3.52$ (A1)