AI SL Practice Set 3 Paper 1 Solution

1. (a) $60300000 A1 N1

[1]

(b) $\$6.03\times10^{7}$ A2 N2

[2]

(c) The percentage error

$=\left|\frac{60300000-61204500}{61204500}\right|\times100\%$ (A1) for substitution

$=1.477832512\%$

$=1.48\%$ A1 N2

[2]

2. (a) The coordinates of the mid-point

$=\left(\frac{3+9}{2},\frac{5+7}{2}\right)$ (A1) for substitution

$=(6, 6)$ A1 N2

[2]

(b) The gradient of L

$=\frac{7-5}{9-3}$ (A1) for substitution

$=\frac{1}{3}$ A1 N2

[2]

(c) The equation of L:

$y-5=\frac{1}{3}(x-3)$ (A1) for substitution

$y-5=\frac{1}{3}x-1$

$y=\frac{1}{3}x+4$ A1 N2

[2]

3. (a) $260-100=(31-11)d$ (M1) for valid approach

$160=20d$

$d=8$

Thus, the common difference is 8. A1 N2

[2]

(b) $u_{11}=100$

$\therefore u_1+(11-1)(8)=100$ (A1) for correct equation

$u_1=20$ A1 N2

[2]

(c) S_{51}

$=\frac{51}{2}\left[2(20)+(51-1)(8)\right]$ (A1) for substitution

$=11220$ A1 N2

[2]

4. (a) 4 A1 N1

[1]

(b) The inter-quartile range

$=6-2.5$ (M1) for valid approach

$=3.5$ A1 N2

[2]

(c) The required probability

$=\frac{8}{12}$ (M1) for valid approach

$=\frac{2}{3}$ A1 N2

[2]

5. (a) The common ratio

$$=\sqrt{\frac{20}{9}\div 20}$$ (M1) for valid approach

$$=\frac{1}{3}$$ A1 N2

[2]

(b) $\frac{20}{81}$ A1 N1

[1]

(c) $S_n=\frac{65600}{2187}$

$$\therefore \frac{20\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}=\frac{65600}{2187}$$ (A1) for correct equation

$$30\left(1-\left(\frac{1}{3}\right)^n\right)-\frac{65600}{2187}=0$$ (A1) for correct approach

By considering the graph of

$y=30\left(1-\left(\frac{1}{3}\right)^n\right)-\frac{65600}{2187}$, $n=8$. A1 N3

[3]

6. (a) $P(X=1)+P(X=2)+P(X=3)+P(X=4)=1$ M1

$\therefore 5k^2+(k^2+6k)+(k^2+k)+k^2=1$ A1

$8k^2+7k-1=0$

$(k+1)(8k-1)=0$ A1

$k=-1$ (*Rejected*) or $k=\frac{1}{8}$ AG N0

[3]

(b) $P(X=2\,|\,X\le 2)$

$=\frac{P(X=2\cap X\le 2)}{P(X\le 2)}$

$=\frac{P(X=2)}{P(X\le 2)}$ (M1) for valid approach

$=\frac{\left(\frac{1}{8}\right)^2+6\left(\frac{1}{8}\right)}{5\left(\frac{1}{8}\right)^2+\left(\left(\frac{1}{8}\right)^2+6\left(\frac{1}{8}\right)\right)}$ (A1) for substitution

$=\frac{49}{54}$ A1 N3

[3]

7. (a) (i) $\begin{cases}15a+7b+2c=97\\3a+5b+9c=99\\4a+4c=48\end{cases}$ A2 N2

(ii) $a=4$, $b=3$ and $c=8$ A3 N3

[5]

(b) $\$248$ A1 N1

[1]

8. (a) $h = -\dfrac{b}{2a}$

$\therefore -5 = -\dfrac{10}{2a}$ (A1) for correct equation

$-5 = -\dfrac{5}{a}$

$a = 1$ A1 N2

[2]

(b) $0 = (-8)^2 + 10(-8) + c$ (M1) for setting equation

$c = 16$ A1 N2

[2]

(c) $\{y : y \geq -9,\ y \in \mathbb{R}\}$ A1 N1

[1]

9. (a) $\cos \mathrm{A\hat{C}B} = \dfrac{\mathrm{AC}^2 + \mathrm{BC}^2 - \mathrm{AB}^2}{2(\mathrm{AC})(\mathrm{BC})}$ (M1) for cosine rule

$\cos \mathrm{A\hat{C}B} = \dfrac{54^2 + 54^2 - 35^2}{2(54)(54)}$ (A1) for substitution

$\cos \mathrm{A\hat{C}B} = 0.789951989$

$\mathrm{A\hat{C}B} = 37.81897498^\circ$

$\mathrm{A\hat{C}B} = 37.8^\circ$ A1 N3

[3]

(b) The required area

$= \dfrac{1}{2}(\mathrm{AC})(\mathrm{BC}) \sin \mathrm{A\hat{C}B}$ (M1) for area formula

$= \dfrac{1}{2}(54)(54) \sin 37.81897498^\circ$ (A1) for substitution

$= 893.999965\ \mathrm{cm}^2$

$= 894\ \mathrm{cm}^2$ A1 N3

[3]

10. (a) $\dfrac{dy}{dx}$

$= \dfrac{1}{4}(4x^3) + 2(2x) + 0$ (A1) for correct derivatives

$= x^3 + 4x$ A1 N2

[2]

(b) The gradient of the tangent at Q

$= 2^3 + 4(2)$ (M1) for substitution

$= 16$ A1 N2

[2]

(c) The equation of the tangent at Q:

$y - 15 = 16(x - 2)$ (M1) for substitution

$y - 15 = 16x - 32$

$16x - y - 17 = 0$ A1 N2

[2]

11. (a) $y = 5$ A1 N1

[1]

(b) (i) $\left(5, \dfrac{7}{2}\right)$ A1 N1

(ii) $k(5) + 2\left(\dfrac{7}{2}\right) - 47 = 0$ (M1) for substitution

$5k = 40$

$k = 8$ A1 N2

(iii) $8x + 2(5) - 47 = 0$ (M1) for substitution

$8x = 37$

$x = \dfrac{37}{8}$

Thus, the required coordinates are

$\left(\dfrac{37}{8}, 5\right)$. A1 N2

[5]

12. (a) $y=\frac{8}{7}$ A2 N2

[2]

(c) $\left\{y: y\neq\frac{8}{7}, y\in\mathbb{R}\right\}$ A1 N1

[1]

(d) $f(x)>g(x)$

$\frac{1-8x}{2-7x}>\frac{1}{2}x^2$

$\frac{1-8x}{2-7x}-\frac{1}{2}x^2>0$ M1

By considering the graph of $y=\frac{1-8x}{2-7x}-\frac{1}{2}x^2$,

$-1.439727<x<0.1239131$ or $\frac{2}{7}<x<1.6015283$.

$\therefore -1.44<x<0.124$ or $\frac{2}{7}<x<1.60$ A2 N3

[3]

13. (a) Let $r\%$ be the nominal annual interest rate compounded yearly.

$(1+r\%)^6=\left(1+\frac{9}{(100)(12)}\right)^{(12)(6)}$ (A1) for substitution

$1+r\%=1.0075^{12}$

$r=9.380689767$ (A1) for correct value

The real interest rate per year

$=9.380689767\%-i\%$

$=(9.38069-i)\%$ A1 N3

[3]

(b) $89000\left(1+\frac{9.38069-i}{100}\right)^6=118000$ (M1) for setting equation

$89000\left(1+\frac{9.38069-i}{100}\right)^6-118000=0$ (A1) for correct approach

By considering the graph of

$y=89000\left(1+\frac{9.38069-i}{100}\right)^6-118000$,

$i=4.5676461$.

Thus, $i=4.57$. A1 N3

[3]

14. (a) 0.0707 A1 N1

[1]

(b) $P(H > q) = 0.37$ (M1) for valid approach

$P(H < q) = 0.63$

$q = 6.225660279$

$q = 6.23$ A1 N2

[2]

(c) $P(6 - t < H < 6 + t) = 0.8$ (M1) for valid approach

$P(H < 6 - t) = 0.1$

$6 - t = 5.128544935$

$t = 0.8714550653$

$t = 0.871$ A1 N2

[2]