AI SL Practice Set 2 Paper 2 Solution

1. (a) (i) $\bar{x}=30000$ A1 N1

(ii) $\bar{y}=9980$ A1 N1

(iii) $a=-0.176$ A1 N1

(iv) $b=15260$ A1 N1

(v) $r=-0.9809315165$ (A1) for correct value
$r=-0.981$ A1 N2

[6]

(b) The estimated insurance cost
$=-0.176(32500)+15260$ (A1) for substitution
$=\$9540$ A1 N2

[2]

(c) The data 52500 km is outside the range of values of x. R1 N1

[1]

(d) (i) H_0: The insurance cost follows the assigned distribution. A1 N1

(ii) $p\text{-value}=0.1031478315$ (A1) for correct value
$p\text{-value}=0.103$ A1 N2

(iii) The null hypothesis is not rejected. A1
As $p\text{-value}>0.05$. R1 N2

[5]

2. (a) $7(98)+24f-2990=0$ — (M1) for setting equation

$24f=2304$

$f=96$ — A1 N2

[2]

(b) $-\frac{7}{24}$ — A1 N1

[1]

(c) (i) The gradient of DE

$=-1\div-\frac{7}{24}$ — (M1) for valid approach

$=\frac{24}{7}$ — A1 N2

(ii) The equation of DE:

$y-10=\frac{24}{7}(x-125)$ — M1A1

$7y-70=24(x-125)$ — A1

$7y-70=24x-3000$

$24x-7y-2930=0$ — AG N0

[5]

(d) $(146, 82)$ — A2 N2

[2]

(e) The coordinates of the mid-point of CD

$=\left(\frac{50+146}{2}, \frac{110+82}{2}\right)$ — M1A1

$=(98, 96)$

Thus, F is the mid-point of CD. — AG N0

[2]

(f) The length of DE

$=\sqrt{(146-125)^2+(82-10)^2}$ — (A1) for substitution

$=75$ — A1 N2

[2]

(g) The area of the triangle CDE

$=\frac{(75)(100)}{2}$ — (M1) for valid approach

$=3750\text{ m}^2$ — A1 N2

[2]

(h) The total area

$= 3750 + \frac{(BC + AE)(AB)}{2}$ (M1)(A1) for correct approach

$= 3750 + \frac{(40 + 115)(100)}{2}$ (A1) for substitution

$= 11500 \text{ m}^2$ A1 N4

[4]

3. (a) $H_1:\ \mu_1 > \mu_2$ A1 N1

[1]

(b) $p\text{-value} = 0.0231895114$ (A1) for correct value

$p\text{-value} = 0.0232$ A1 N2

[2]

(c) The null hypothesis is rejected. A1

As $p\text{-value} < 0.05$. R1 N2

[2]

(d) (i) The required probability

$= \left(\frac{5}{10}\right)\left(\frac{2}{9}\right)$ (A1) for correct formula

$= \frac{1}{9}$ A1 N2

(ii) The required probability

$= \left(\frac{5}{10}\right)\left(\frac{2}{9}\right) + \left(\frac{5}{10}\right)\left(\frac{7}{9}\right) + \left(\frac{5}{10}\right)\left(\frac{2}{9}\right)$ (A1) for correct formula

$= \frac{11}{18}$ A1 N2

[4]

(e) H_1: The age and the reading preference are not independent. A1 N1

[1]

(f) 4 A1 N1

[1]

(g) $\chi^2_{calc} = 53.64204545$ (A1) for correct value

$\chi^2_{calc} = 53.6$ A1 N2

[2]

(h) The null hypothesis is rejected. A1

As $\chi^2_{calc} > 13.277$. R1 N2

[2]

4. (a) $AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos A\hat{B}C$ (M1) for cosine rule

$AC^2 = 15^2 + 13.5^2 - 2(15)(13.5)\cos 98^\circ$ (A1) for substitution

$AC = 21.53172324\text{ m}$

$AC = 21.5\text{ m}$ A1 N3

[3]

(b) $\dfrac{\sin B\hat{A}C}{BC} = \dfrac{\sin A\hat{B}C}{AC}$ (M1) for sine rule

$\dfrac{\sin B\hat{A}C}{13.5} = \dfrac{\sin 98^\circ}{21.53172324}$ (A1) for substitution

$\sin B\hat{A}C = \dfrac{13.5\sin 98^\circ}{21.53172324}$

$B\hat{A}C = 38.38043409^\circ$

$B\hat{A}C = 38.4^\circ$ A1 N3

[3]

(c) The area of the triangular region ABC

$= \dfrac{1}{2}(AB)(BC)\sin A\hat{B}C$ (M1) for area formula

$= \dfrac{1}{2}(15)(13.5)\sin 98^\circ$ (A1) for substitution

$= 100.264642\text{ m}^2$

$= 100\text{ m}^2$ A1 N3

[3]

(d) The height of the vertical pole VA

$= 15\tan 22.1^\circ$ (M1) for valid approach

$= 6.090868387\text{ m}$ (A1) for correct value

Let θ be the required angle of depression.

$\tan\theta = \dfrac{6.090868387}{21.53172324}$ (M1) for valid approach

$\theta = 15.79508441^\circ$

Thus, the angle of depression of C from V is 15.8°. A1 N4

[4]

5. (a) $f'(x) = -3x^2 + b(2x) - 432(1) + 0$ (A1) for correct derivatives

$f'(x) = -3x^2 + 2bx - 432$

$f'(8) = 0$ (M1) for setting equation

$\therefore -3(8)^2 + 2b(8) - 432 = 0$ (A1) for substitution

$16b = 624$

$b = 39$ A1 N4

[4]

(b) (i) 984 A1 N1

(ii) $(18, 1484)$ A2 N2

[3]

(c) $8 < x < 18$ A2 N2

[2]

(d) (i) $984 < k < 1484$ A2 N2

(ii) $k \le 984$ or $k \ge 1484$ A2 N2

[4]

(e) $C(x) = -x^3 + 39x^2 - 432x + 2456$

$C(8) = 984$

$C(25)$

$= -25^3 + 39(25)^2 - 432(25) + 2456$ A1

$= 406$

$C(8) > C(25)$ R1

Thus, the average cost attains its minimum when 25000 smart watches are produced. AG N0

[2]

(f) $C(x) \le 984$ (M1) for setting inequality

$-x^3 + 39x^2 - 432x + 2456 \le 984$

$-x^3 + 39x^2 - 432x + 1472 \le 0$

By considering the graph of

$y = -x^3 + 39x^2 - 432x + 1472$, $x = 8$ or $x \ge 23$.

Thus, the range of values of x are $x = 8$ or $23 \le x \le 25$. A2 N3

[3]