Chapter 15 Solution

Exercise 63

1. $\int_3^{10} \frac{5}{5x-1}\,dx$

$= \left[\frac{1}{5}\times 5\ln(5x-1)\right]_3^{10}$	A2
$= \ln(5(10)-1) - \ln(5(3)-1)$	(M1) for substitution
$= \ln 49 - \ln 14$	A1
$= \ln\frac{49}{14}$	(A1) for correct formula
$= \ln\frac{7}{2}$	
$\therefore k = \frac{7}{2}$	A1 N3

[6]

2. $\int_0^{6} \frac{4}{4x+1}\,dx$

$= \left[\frac{1}{4}\times 4\ln(4x+1)\right]_0^{6}$	A2
$= \ln(4(6)+1) - \ln(4(0)+1)$	(M1) for substitution
$= \ln 25$	A1
$= \ln 5^2$	
$= 2\ln 5$	(A1) for correct formula
$\therefore k = 5$	A1 N3

[6]

3. $\int_0^k \frac{1}{3x+4}\,dx$

$= \left[\frac{1}{3} \times \ln(3x+4)\right]_0^k$ A1

$= \frac{1}{3}\ln(3k+4) - \frac{1}{3}\ln(3(0)+4)$ (M1) for substitution

$= \frac{1}{3}\ln\frac{3k+4}{4}$ A1

$\frac{1}{3}\ln\frac{3k+4}{4} = \ln 2$ (M1) for setting equation

$\ln\frac{3k+4}{4} = 3\ln 2$

$\ln\frac{3k+4}{4} = \ln 2^3$

$\frac{3k+4}{4} = 8$ M1

$3k+4 = 32$

$k = \frac{28}{3}$ A1 N3

[6]

4. $\int_0^k \frac{1}{9-x}\,dx$

$= \left[\frac{1}{-1} \times \ln(9-x)\right]_0^k$ A1

$= -[\ln(9-k) - \ln(9-0)]$ (M1) for substitution

$= \ln\frac{9}{9-k}$ A1

$\ln\frac{9}{9-k} = \ln\frac{k}{2}$ (M1) for setting equation

$\frac{9}{9-k} = \frac{k}{2}$ M1

$18 = 9k - k^2$

$k^2 - 9k + 18 = 0$

$(k-3)(k-6) = 0$

$k = 3$ or $k = 6$ A2 N4

[7]

Exercise 64

1. $f(x)=\int 3x^2(x^3+1)^6\,dx$ (M1) for indefinite integral

Let $u=x^3+1$. A1

$\frac{du}{dx}=3x^2 \Rightarrow du=3x^2dx$

$\therefore f(x)$

$=\int u^6du$

$=\frac{1}{7}u^7+C$

$=\frac{1}{7}(x^3+1)^7+C$ A1

$2=\frac{1}{7}((-1)^3+1)^7+C$ M1

$C=2$ (A1) for correct value

$\therefore f(x)=\frac{1}{7}(x^3+1)^7+2$ A1 N4

[6]

2. $f(x)=\int 2x\sin(x^2)dx$ (M1) for indefinite integral

Let $u=x^2$. A1

$\frac{du}{dx}=2x \Rightarrow du=2xdx$

$\therefore f(x)$

$=\int \sin u\,du$

$=-\cos u+C$

$=-\cos(x^2)+C$ A1

$-1=-\cos(0)^2+C$ M1

$C=0$ (A1) for correct value

$\therefore f(x)=-\cos(x^2)$ A1 N4

[6]

3. $f(x) = \int \cos^3 2x \sin 2x \mathrm{d}x$ (M1) for indefinite integral

Let $u = \cos 2x$. A1

$\frac{\mathrm{d}u}{\mathrm{d}x} = -2\sin 2x \Rightarrow -\frac{1}{2}\mathrm{d}u = \sin 2x \mathrm{d}x$ A1

$\therefore f(x)$

$= \int -\frac{1}{2}u^3 \mathrm{d}u$

$= -\frac{1}{8}u^4 + C$

$= -\frac{1}{8}\cos^4 2x + C$ A1

$3 = -\frac{1}{8}\cos^4\left(2\left(\frac{\pi}{2}\right)\right) + C$ M1

$C = \frac{25}{8}$ (A1) for correct value

$\therefore f(x) = -\frac{1}{8}\cos^4 2x + \frac{25}{8}$ A1 N4

[7]

4. $f(x) = \int 4x^3 e^{x^4} \mathrm{d}x$ (M1) for indefinite integral

Let $u = x^4$. A1

$\frac{\mathrm{d}u}{\mathrm{d}x} = 4x^3 \Rightarrow \mathrm{d}u = 4x^3 \mathrm{d}x$ A1

$\therefore f(x)$

$= \int e^u \mathrm{d}u$

$= e^u + C$

$= e^{x^4} + C$ A1

$e^{16} - 1 = e^{2^4} + C$ M1

$C = -1$ (A1) for correct value

$\therefore f(x) = e^{x^4} - 1$ A1 N4

[7]

Exercise 65

1. (a) $\int_9^1 3f(x)\mathrm{d}x$

$=3\int_9^1 f(x)\mathrm{d}x$ A1

$=-3\int_1^9 f(x)\mathrm{d}x$ A1

$=-3(10)$

$=-30$ AG N0

[2]

(b) $\int_7^9 (x+f(x))\mathrm{d}x+\int_1^7 (x+f(x))\mathrm{d}x$

$=\int_1^9 (x+f(x))\mathrm{d}x$ (A1) for combining integrals

$=\int_1^9 x\mathrm{d}x+\int_1^9 f(x)\mathrm{d}x$ (A1) for separating integrals

$=\left[\frac{1}{2}x^2\right]_1^9+10$ A1

$=\frac{1}{2}(9)^2-\frac{1}{2}(1)^2+10$ A1

$=50$ A1 N3

[5]

2. (a) $\int_{10}^3 5f(x)\mathrm{d}x$

$=5\int_{10}^3 f(x)\mathrm{d}x$ A1

$=-5\int_3^{10} f(x)\mathrm{d}x$ A1

$=-5(-4)$

$=20$ AG N0

[2]

(b) $\int_5^{10} (x+2f(x))\mathrm{d}x+\int_3^5 (x+2f(x))\mathrm{d}x$

$=\int_3^{10} (x+2f(x))\mathrm{d}x$ (A1) for combining integrals

$=\int_3^{10} x\mathrm{d}x+2\int_3^{10} f(x)\mathrm{d}x$ (A1) for separating integrals

$=\left[\frac{1}{2}x^2\right]_3^{10}+2(-4)$ A1

$=\frac{1}{2}(10)^2-\frac{1}{2}(3)^2-8$ A1

$=\frac{75}{2}$ A1 N3

[5]

3. (a) $\int_6^0 f(x)\mathrm{d}x$

$= \frac{1}{4}\int_6^0 4f(x)\mathrm{d}x$ A1

$= -\frac{1}{4}\int_0^6 4f(x)\mathrm{d}x$ A1

$= -\frac{1}{4}(12)$

$= -3$ AG N0

[2]

(b) $\int_5^6 (x^2 + f(x))\mathrm{d}x + \int_0^5 (x^2 + f(x))\mathrm{d}x$

$= \int_0^6 (x^2 + f(x))\mathrm{d}x$ (A1) for combining integrals

$= \int_0^6 x^2\mathrm{d}x + \int_0^6 f(x)\mathrm{d}x$ (A1) for separating integrals

$= \left[\frac{1}{3}x^3\right]_0^6 - 3$ A1

$= \frac{1}{3}(6)^3 - \frac{1}{3}(0)^2 - 3$ A1

$= 69$ A1 N3

[5]

4. (a) $\int_2^1 4f(x)\,dx$

$= \frac{4}{3}\int_2^1 3f(x)\,dx$ A1

$= -\frac{4}{3}\int_1^2 3f(x)\,dx$ A1

$= -\frac{4}{3}(6)$

$= -8$ AG N0

[2]

(b) $\int_{1.3}^1 \left(\frac{1}{x}+3f(x)\right)dx + \int_2^{1.3}\left(\frac{1}{x}+3f(x)\right)dx$

$= \int_2^1 \left(\frac{1}{x}+3f(x)\right)dx$ (A1) for combining integrals

$= -\int_1^2 \left(\frac{1}{x}+3f(x)\right)dx$

$= -\int_1^2 \frac{1}{x}dx - 3\int_1^2 f(x)\,dx$ (A1) for separating integrals

$= -[\ln x]_1^2 - 6$ A1

$= -\ln 2 + \ln 1 - 6$ A1

$= -\ln 2 - 6$ A1 N3

[5]

Exercise 66

1.	(a)	$f'(x)=0$	(M1) for setting equation
		$3x^2-2x-8=0$	
		$(3x+4)(x-2)=0$	M1A1
		$x=2$ or $x=-\frac{4}{3}$ (*Rejected*)	A1
		$\therefore x=2$	A1 N2
			[5]
	(b)	$f(x)$	
		$=\int(3x^2-2x-8)\mathrm{d}x$	(M1) for indefinite integral
		$=x^3-x^2-8x+C$	A3
		$7=0^3-0^2-8(0)+C$	
		$C=7$	(A1) for correct value
		$\therefore f(x)=x^3-x^2-8x+7$	A1 N3
			[6]
	(c)	$g(x)=-f(x+3)-4$	(A1) for transformation
		The local minimum point on the graph of g is the image of A .	(M1) for recognizing image
		The x-coordinate of the required point	
		$=2-3$	M1
		$=-1$	A1 N4

2.	(a)	$f'(x)=0$	(M1) for setting equation
		$36-x^2=0$	
		$(6+x)(6-x)=0$	M1A1
		$x=6$ or $x=-6$ (*Rejected*)	A1
		$\therefore x=6$	A1 N2
			[5]
	(b)	$f(x)$	
		$=\int(36-x^2)\mathrm{d}x$	(M1) for indefinite integral
		$=36x-\frac{1}{3}x^3+C$	A3
		$6=36(0)-\frac{1}{3}(0)^3+C$	
		$C=6$	(A1) for correct value
		$\therefore f(x)=36x-\frac{1}{3}x^3+6$	A1 N3
			[6]
	(c)	$g(x)=f(-(x-6))+5$	(A1) for transformation
		The local maximum point on the graph of g is the image of A .	(M1) for recognizing image
		The x-coordinate of the required point	
		$=-6+6$	M1
		$=0$	A1 N4
			[4]

3. (a)

$f(x)$	
$=\int (x^2-9)\mathrm{d}x$	(M1) for indefinite integral
$=\frac{1}{3}x^3-9x+C$	A2
$0=\frac{1}{3}(0)^3-9(0)+C$	(M1) for substitution
$C=0$	(A1) for correct value
$\therefore f(x)=\frac{1}{3}x^3-9x$	A1 N4

[6]

(b)

$f'(x)=0$	(M1) for setting equation
$x^2-9=0$	
$(x+3)(x-3)=0$	A1
$x=-3$ (*Rejected*) or $x=3$	M1A1
When $x=3$, $y=\frac{1}{3}(3)^3-9(3)=-18$	M1
Thus, the coordinates of A are $(3,-18)$.	A1 N2

[6]

(c)

$g(x)=2f(x-1)+4$	(A1) for transformation
The local minimum point on the graph of g is the image of A.	(M1) for recognizing image
The coordinates of the required point	
$=(3+1,\ 2(-18)+4)$	(M1) for valid approach
$=(4,-32)$	A1 N4

[4]

4. (a) $f(x)$

$= \int(-2x-4)\mathrm{d}x$ — (M1) for indefinite integral

$= -x^2 - 4x + C$ — A2

$-5 = -(1)^2 - 4(1) + C$ — (M1) for substitution

$C = 0$ — (A1) for correct value

$\therefore f(x) = -x^2 - 4x$ — A1 N4

[6]

(b) $f'(x) = 0$ — (M1) for setting equation

$-2x - 4 = 0$

$x = -2$ — A1

When $x = -2$, $y = -(-2)^2 - 4(-2) = 4$ — M1

Thus, the coordinates of A are $(-2, 4)$. — A1 N2

[4]

(c) $g(x) = -f(3x)$ — (A1) for transformation

The local minimum point on the graph of g is the image of A. — (M1) for recognizing image

The coordinates of the required point

$= (-2 \div 3, -4)$ — (M1) for valid approach

$= \left(-\frac{2}{3}, -4\right)$ — A1 N4

[4]